Zakład Kompatybilności Elektro-magnetycznej
ul. Swojczycka 38
51-501 Wrocław
T:[+71] 36 99 803,
F:[+71] 37 28 8788
www.itl.waw.pl
sekretariat@il.wroc.pl

INSTYTUT ŁĄCZNOŚCI
PAŃSTWOWY INSTYTUT BADAWCZY

National Institute of Telecommunications
ul. Szachowa 1
PL – 04-894 Warszawa
T: [+48 2] 512 81 00
F: [+48 22] 512 86 25
E-mail: info@itl.waw.pl
www.itl.waw.pl

Opracowanie oraz analiza efektywności algorytmów ESPRIT oraz Root-MUSIC w celu ustalenia parametrów kanału OFDM

Adjustment and Efficiency Analysis of the Algorithms ESPRIT and Root-MUSIC for Parameters Estimation of the OFDM Channel

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Kierownik Zakładu: dr inż. Janusz Sobolewski

Contents:

1. Introduction

The OFDM modulated signals are basis for digital broadcasting systems DRM [1], EUREKA-147 DAB [2], DVB-T [3], some versions of WLAN projects [4] and very probably the planned wireless system 4G [5]. The main advantages in applications of the OFDM coder are its low complexity and the comparatively low cost with regard to competitive CDMA coder. Application of the OFDM modulation allows avoiding the inter-symbol interference following the multipath in wireless channels. To this aim the useful parts T_u of the consecutive OFDM symbols are separated by a guard interval T_g of a length smaller then time delay spread of the channel. The length of guard interval influences the throughput of the system. Setting this parameter constant means the non-optimal conditions of transmission.

An example of the OFDM system where guard interval can be chosen in adaptive way is the DRM system [1]. In DRM system the guard interval length is a parameter with four different options depending on 'robust level'. The length of the whole symbol (T_g + T_U) remains constant. In regions where the propagation conditions are changing in time the optimal quality of transmission demands adaptive changes of the transmission parameters. This is not possible without periodic control of the time delay spread of the signal.

The cheapest way of the delay spread control is the software estimation of the individual paths time delays. As was pointed out in [6] an every algorithm of the Direction-On-Arrival problem can be theoretically used for time delay estimations in OFDM channels. The practical usefulness of these methods for the OFDM signal estimations and limits of possible evaluation are still not recognized.

Among the different DOA algorithms which were developed for applications in the area of seismic exploration, radar, sonar, electronic surveillance, robotics and system control - the methods of high-resolution frequency estimation include basic algorithms:

- MUSIC (Multiple Signal Classification)
- Root-MUSIC
- ESPRIT (Estimation of Signal Parameters via Rotational Invariance Transformation/ Techniques)

All the three methods take advantage of the eigendecomposition of the covariance matrix of the signal probes in time periods and next the extraction of the noise subspace in a space stretched by eigenvectors of covariance matrix.

The following steps of the algorithms substantially differ.

In application to the OFDM time delay estimation there appear problems of type different then those in the DOA area:

1. In practical applications the number of paths in OFDM signals is higher then that in DOA problem cases, where normally only three paths are taken into account. The OFDM signals include more paths and there can appear problem of method resolution

2. In problems of DOA it is assumed that different signal paths are not correlated. In case of the OFDM signal the paths are coherent as:
   i. reflected signals
   ii. different transmitter's signals in Single Frequency Networks.

3. In case of coherent signals there is necessity of providing the procedure of preprocessing called the (frequency) smoothing. The smoothing aims at the signal decorrelation and the signal noise mitigation. In the case of algorithm ESPRIT there appears the demand of the analysis of the compatibility of smoothing preprocessing with the rotational invariance of the method

4. The range of parameters to be taken into account in the estimation of the OFDM signal include three different classes:
   i. Parameters of the channel
   ii. Parameters of the signal
   iii. Parameters of the algorithm

   Some of these parameters are common in different classes as e.g. the OFDM symbol frequency block and the OFDM channel wideband

The OFDM channel estimation - including signal time delay control - is the problem not only in the classical SFN networks but also the important part of the MIMO channel estimations deciding about the demodulation accuracy in future wireless systems.

1.1. On analogy between description of the DOA and OFDM systems

The operations of HF demodulation and FFT transformations on received baseband OFDM signal with $\tau \leq T_g$ lead to a set of subcarrier factors:

$$(S_k^{(n)})^{-1} \cdot R_k^{(n)} = \sum_{i=0}^{d-1} e^{-j2\pi\cdot(k-1)\frac{\tau_i}{T_U}} \cdot A_{i,1}^{(n)} + N_k^{(n)} \tag{1.1}$$

for k = 1, 2, …, K. The complex amplitudes $\mathbf{A}_i$ depend on τ_i through expression not depending on subcarrier index *k*:

$$\mathbf{A}_i = A_i \cdot e^{j[\theta_i - (\omega_c + \omega_i^D)\tau_i]} \tag{1.2}$$

For pilot subcarriers the modulation factors $\{S_k^{(n)}\}$are known a priori and the set of equations (1.1) appears of Vandermonde type. This type of equations is equivalent to set of equations describing the signals in linear array of sensors in Direction-of-Arrival problem

Presented analogy is illustrated in Fig. 1. The role of DOA series of signal probes in time in case of the OFDM estimation play the demodulation factors in pilot subcarriers of the OFDM block.

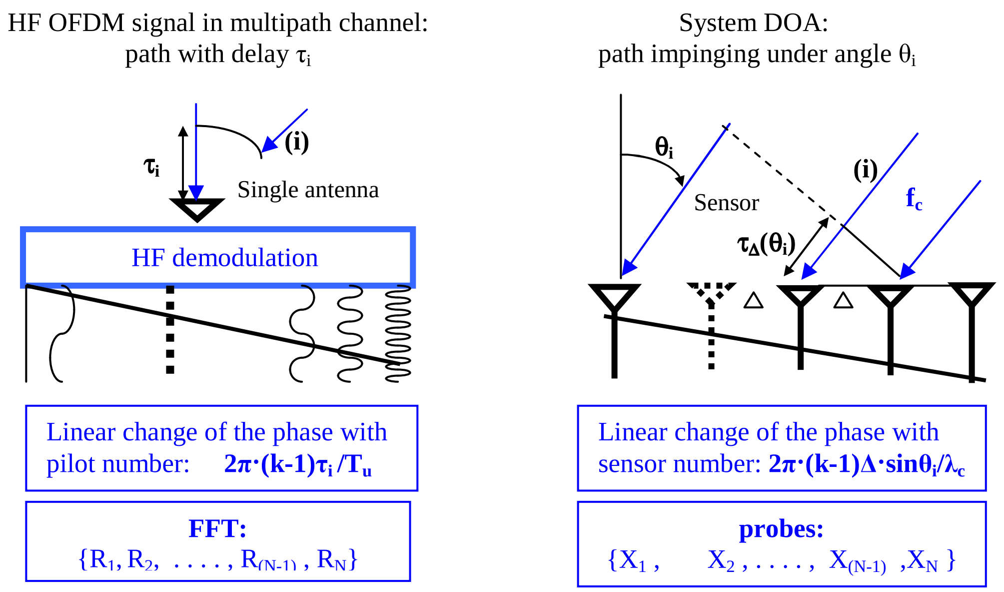


Fig.1. An illustration of analogy between DOA and OFDM problems

The well recognized theory of DOA algorithms leads to conclusions that OFDM system is formally equivalent to the sensor array of the type:

- linear
- with equally spaced sensors
- with omni directional sensor characteristics

1.2. The OFDM demodulation factors for time delay over the guard interval: $T_g < \tau < Tu$. Extension to the Vandermonde equations

When paths with delays outside guard interval can appear, i.e. $\tau > T_g$ are allowed, in useful field processed in FFT demodulation enters discontinuity between (n-1) and n-th OFDM symbols. This leads to the intercarrier interference. The set (1.1) in this case is replaced by the set of equations [6]:

$$R_k^{(n/n-1)} = \sum_{\tau_i \le T_g} S_k^{(n)} A_{i,1}^{(n)} \cdot e^{-j2\pi \frac{k-1}{T_U}\tau_i} +$$
$$\sum_{\tau_j > T_g} \left\{ S_k^{(n)} \left[\frac{T_U - \Delta\tau_j}{T_U} A_{j,1}^{(n)} \right] \cdot e^{-j2\pi \frac{k-1}{T_U}\tau_j} + S_k^{(n-1)} \left[\frac{\Delta\tau_i}{T_U} A_{j,1}^{(n-1)} \right] \cdot e^{-j2\pi \frac{k-1}{T_U}(\tau_j - T_g)} \right\} +$$
$$+ \frac{1}{N} \sum_{\tau_j > T_g} \sum_{\substack{r=1 \\ r \ne k}}^{N} \frac{1 - e^{j2\pi \frac{r-k}{T_U}(\tau_j - T_g)}}{1 - e^{j2\pi \frac{r-k}{N}}} \left\{ S_r^{(n-1)} A_{j,1}^{(n-1)} e^{-j2\pi \frac{r-1}{T_U}(\tau_j - T_g)} - S_r^{(n)} A_{j,1}^{(n)} e^{-j2\pi \frac{r-1}{T_U}\tau_j} \right\} + N_k \qquad (1.3)$$

In formula (1.3) the the k-th modulating factor depends not only on modulation factors $S_k^{(n)}$ of the k-th pilot in n-th symbol, but also on modulation factors of all other subcarriers in the n and (n-1) OFDM symbols. Without knowing these factors it is not possible to draw further information from the set (1.3). The set of equations (1.3) can be reduced to the form:

$$R_k^{(n/n-1)} = \sum_{\tau_i < T_U} B_i^{n,(n-1)} \cdot e^{-j2\pi \frac{k-1}{T_U}\tau_i} + N_k \qquad (1.4)$$

only in case when analyzed symbols n and (n-1) contain all subcarriers with known modulation factors. In the following it is assumed that $S_k \equiv 1$ for k = 1, 2, …, K, where K is a number of pilot subcarriers in analyzed OFDM symbol.

The range of time delays in formula (1.4) can spread within limits 0 – Tu. This means that estimations of time delays can be provided for time spread over guard interval up to the length of the useful part of the OFDM symbol.

In such case however equations (1.4) do not allow to calculate the signal amplitudes.

2. Algorithms

The basis of both algorithms Root-MUSIC and ESPRIT is the algorithm MUSIC.

2.1. Algorithm MUSIC

The basic idea of the algorithm MUSIC (ang. MUltiple SIgnal Classification) in application to the OFDM signals estimation [6] is the spectral decomposition of the covariance matrix of OFDM subcarrier modulation symbols in OFDM blocks. The analysis of the space of eigenvectors of the covariance matrix allows extracting the basis vectors of the noise subspace. The values of time delays $\{\tau_i\}$ can be estimated from the sum of projections of vector $a(\tau)$ on vectors spanning the noise subspace for parameter τ within limits $[0, \tau_{max}]$. The minima of such 'cost function' appoint the searched values of time delays $\{\tau_i\}$.

The fundamental steps of the MUSIC algorithm are presented in Fig. 2.

Among them the first steps are preserved as part of the algorithms Root-MUSIC and ESPRIT:

1. Collect the modulating symbols of the OFDM block
2. Choose the dimension of the correlation matrix (M > d)
3. Perform the smoothing procedure
4. Solve the eigenvalue problem $\mathbf{M}f_i = \lambda_i f_i$
5. Place the eigenvalues into grooving series $\lambda_1 > \lambda_2 > \ldots > \lambda_d > \ldots > \lambda_m$, M =dim **M.**
6. In adequate series place the eigenvectors f_i
7. Extract the noise subspace: $\{f_{d+1,\ldots,}f_m\}$
8. Estimate the number of the delayed paths d

The further steps of the algorithm determine the individual procedures of the MUSIC algorithm:

9. Construct the general steering vector

$$a(\tau_n) = [1, \exp(-i2\pi p\tau/Tu), \ \exp(-i2\pi 2p\tau/Tu), \ldots, \exp(-i2\pi(K-1)p\tau/Tu)]$$

10. Project general steering vector $a(\tau_n)$ on the basic vectors of the noise subspace

$$a(\tau_n)f_i, \ i = 1, 2, \ldots, n$$

11. Calculate the cost function

$$C(n)=1/||a(\tau_n)f||$$

12. Extract the maxima of the cost function

$$\tau_n = \tau_{(n)}, \ n: \max C(n)$$

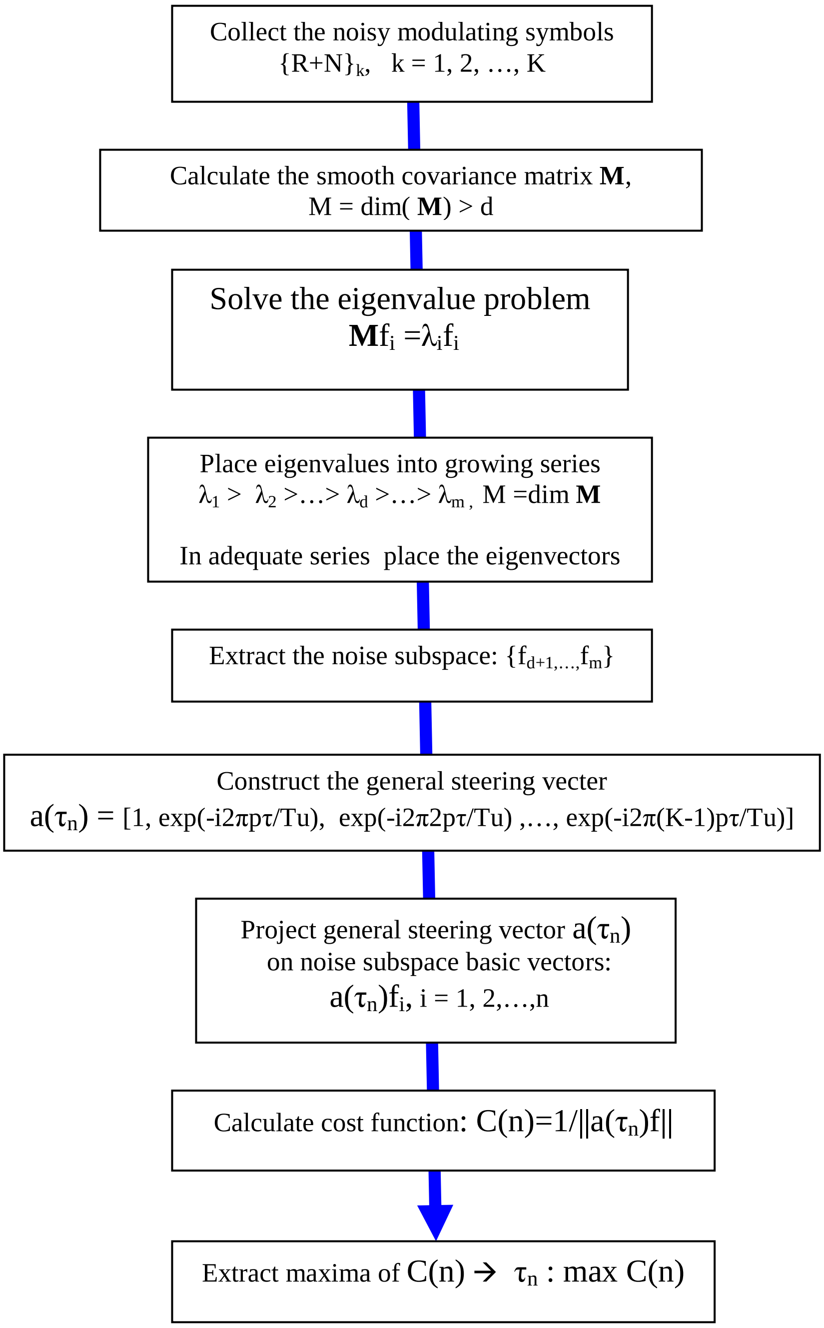


Fig. 2. The main steps of the algorithm MUSIC

2.2. Algorithm Root-MUSIC in application to the OFDM signals

The algorithm Root-MUSIC [7]-[9] takes advantage of the smoothed MUSIC algorithm for calculating vectors spanning the noise subspace, but introduces the new method of estimating signal parameters. In place of the time consuming process of probing the phase space - these parameters are calculated from roots of the linear equation with coefficients constructed from the vectors of the noise subspace [7]. The Root-MUSIC algorithm can be applied in the case when the matrix A is of Vandermonde type.

The steering vector **a**(τ), parameterized by time delay τ, and every vector f_i from the basis spanning the noise subspace should be orthogonal:

$$a^H(\tau) f_i = 0 \qquad /2.1/$$

Here:

$$a(\tau) = [a_1, a_2, ..., a_M], \quad a_k = \exp(-j2\pi\,(\mathrm{k}-1)\mathrm{p}\cdot\tau/\mathrm{T_U}); \qquad \mathrm{f_i} = [\mathrm{f_{i1}}, \mathrm{f_{i2}}, ..., \mathrm{f_{iM}}], \ \ \mathrm{f_{ik}} \in \textit{Noise subspace}$$

and the value of 'p' is equal to the inter-pilot separation distance in the OFDM symbol.

From (2.1) it follows that also the sum of squares of maps of **a**(τ) on individual basic noise vectors should equal zero. So:

$$a^H(\tau) F \cdot F^H a(\tau) = 0 \qquad /2.2/$$

where matrix F is equal $F = [f_{(s+1)}, f_{(s+2)}, \ldots, f_M]$.

Let assign the matrix product FF^H as D:

$$D = FF^H \qquad /2.3/$$

Introducing the complex variable *z*:

$$z(\tau) = \exp(-j2\pi \cdot p\frac{\tau}{T_U}), \qquad /2.4/$$

which fulfills the identity:

$$z^* = z^{-1} \qquad /2.4a/$$

it allows to replace the components of vector **a**(τ), for τ within $[0, T_U)$, by powers of z(τ):

$$a_k(\tau) = \exp(-j2\pi[k-1]p\frac{\tau}{T_U}) = \left[\exp(-j2\pi \cdot p\frac{\tau}{T_U})\right]^{(k-1)} = z(\tau)^{(k-1)} \qquad /2.5/$$

Putting (2.5) into /2.2/ leads to equation:

$$z^{H}(\tau)D\cdot z(\tau)=0, \quad \text{where} \quad z(\tau)\equiv[z^{0}\ z^{1},...,z^{(M\text{-}1)}]^{H} \qquad /2.6/$$

or, in an explicit form (here index τ is abandoned):

$$\sum_{k=1}^{M}\sum_{l=1}^{M}(z^{*})^{k}\ D_{kl}\cdot z^{l}=0 \qquad /2.7/$$

It follows from the definition of the matrix D that D = D^H, or $D_{k,l}$ = $D^*_{l,k}$. So for every solution z_i of equation /2.7/ there also exists alternative complex solution z^*_i .
Taking into account (2.4 a) equation (2.7) can be rewritten in the form:

$$\sum_{k=1}^{M}\sum_{l=1}^{M}z^{-k}\ D_{kl}\cdot z^{l}=\sum_{k=1}^{M}\sum_{l=1}^{M}z^{-(k-l)}\ D_{kl}=0 \qquad /2.8/$$

Introducing in (2.8) a new summation variable n ≡ k-l one gets:

$$\sum_{n=-(M-1)}^{M-1}z^{-n}(\sum_{k-l=n}D_{k,l})=0 \qquad /2.9/$$

If both sides of (2.9) are multipled by $z^{(M-1)}$ (it is assumed that $z \neq 0$) finally we obtain an equation:

$$D_{-(M-1)}+zD_{-(M-2)}+...+z^{M-2}D_{-1}+z^{M-1}D_{0}+z^{M}D_{1}+...+z^{2(M-1)}D_{(M-1)}=0 \qquad /2.10/$$

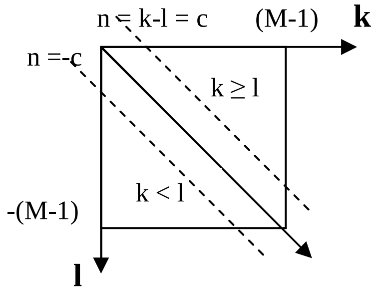
where

$$D_{n}=\sum_{k-l=n}D_{k,l}\ ; \quad n=-(M-1),-(M-2),...,(M-1) \qquad /2.10a/$$

The ***2M-1***coefficients D_n could be calculated from formulae:

$$D_{n}=\begin{cases}\sum_{k=n}^{M-1}D_{k,k-n}=(D_{n,0}+D_{n+1,1}+...+D_{M-1,M-1-n}) & \text{for}\quad n\geq 0\\ \sum_{k=0}^{M-1-|n|}D_{k,|n|+k}=(D_{0,|n|}+D_{1,|n|+1}+...+D_{M-1-|n|,M-1}) & \text{for}\quad n<0\end{cases} \qquad /2.10b/$$

The positions of matrix D entries are presented in Fig. 3**.**



Rys. 3. Distribution of the entries of the matrix D with constant values n = k-l

The hermitean property of the matrix $\{D_{kl}\}$ and equality (2.10) lead to the conclusion that $D_{-n} = D^*_n$.

$$D^*_{(M-1)} + zD^*_{(M-2)} + \ldots + z^{M-2}D^*_1 + z^{M-1}D_0 + z^M D_1 + \ldots + z^{2(M-1)}D_{(M-1)} = 0 \qquad /2.11/$$

Equation (2.11) is a linear complex equation of the rank ***2(M-1)***. In the ideal case, when the noise subspace is orthogonal to the signal subspace, among the roots lying on the circle |r| = 1 in the complex plane only one value is chosen from every complex pair

$$\exp(-j2\pi \cdot p\frac{\tau}{T_U}) \equiv z(\tau) = |z| \cdot e^{j\arg(z)},$$

/2.12/

In situation when vectors f_i do not span exactly the noise subspace the roots z_i need not to lye on the circle. Organizing the roots into a series with growing low modules:

$$1 \geq |z_1| \geq |z_2| \ldots \geq |z_{(2M-1)}|$$

and choosing the L with greatest modules one obtains the values of time delays $\{\tau_i\}$ for i = 1, 2,…, L:

$$\tau_i = \frac{T_U}{2\pi \cdot p}\arg(z_i)$$

/2.13/

or , taking into account the limits of the parameter τ:

$$\tau_i = \mathrm{mod}(\frac{T_U}{2\pi \cdot p}\arg(z_i), T_U) \qquad /2.13a/$$

The basic steps of the algorithm Root-MUSIC in application to the OFDM signal:

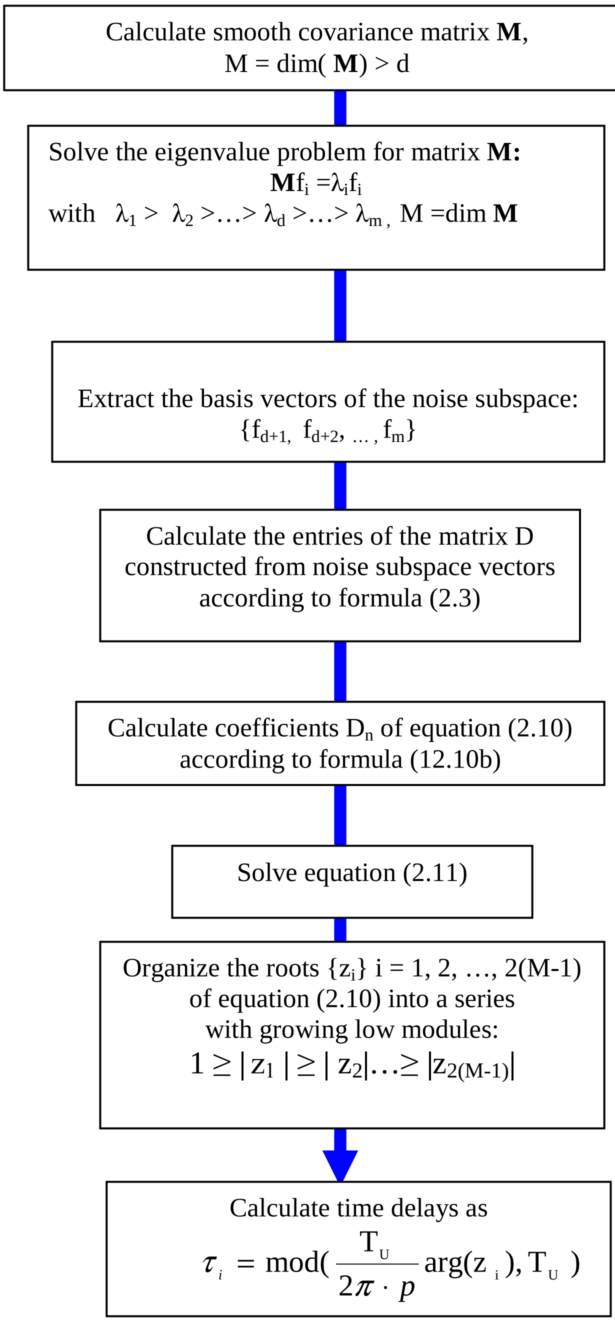


Fig. 4. The main steps of the Root-MUSIC algorithm

Results of computer simulations and following conclusions are placed in Section III A

2.3. *Algorithm ESPRIT in application to the OFDM signals*

Algorithm ESPRIT (ang. Estimation of Signal Parameters via Rotational Invariance technique) takes advantage of the translational symmetry between two subarrays of sensors with identical characteristics. This linear invariance of subarrays leads to the rotational invariance of the subarray vectors in the signal subspace [10] - [11].

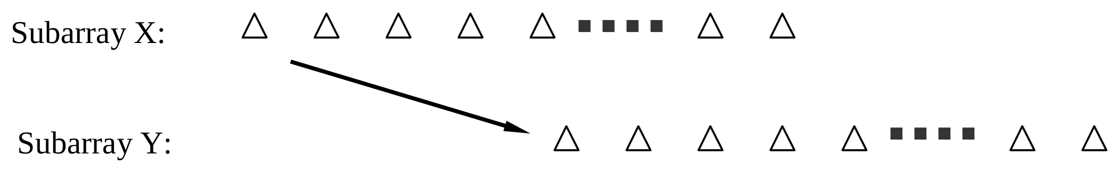


Fig. 5. Translational invariance of subarrays in the sensor network

Assuming that the manifold matrix of one subarray is equal steering matrix **A**, the linear symmetry between both sub-arrays leads to manifold matrix of the second subrray as **A·Φ**, where matrix **Φ** is described by its entries:

$$\mathbf{\Phi}_{kl} = \delta_{kl} \cdot \exp(-j\ 2\pi \cdot \Delta \cdot \sin\theta_i / c) \qquad /3.1a/$$

with δ_{kl} – Dirac's distribution; Δ – length of translation distance; θ_i – an angle of impinging path; c - velocity of light.

In the case of subarrays constructed from subcarriers of the OFDM symbol formula (3.1.a) should be replaced by another one:

$$\mathbf{\Phi}_{kl} = \delta_{kl} \cdot \exp(-j\ 2\pi \cdot \Delta \cdot p \cdot \tau_k / Tu) \qquad /3.1b/$$

As known from MUSIC algorithm the columns of the d-dim steering matrix A are orthogonal to the noise subspace and so span the same space as the d vectors of the signal subspace E_S in the eigenvector decomposition of the covariance matrix R, i.e.:

$$A(\tau) \perp E_N \quad \text{and} \quad E_S \perp E_N$$

Spanning the same d-dim space the vectors E_S and the steering vectors of matrix A have to be related by some linear transform represented by matrix T:

$$E_S = A \cdot T, \qquad (3.2)$$

or, in composite subspaces related with subarrays X and Y:

$$E_X = A_X \cdot T \qquad (3.3a)$$

$$E_Y = A_Y \cdot T = A_X \Phi \cdot T \qquad (3.3b)$$

On the other hand, because E_X and E_Y span the same d-dim signal subspace they are linearly dependent and related by some linear transform F:

$$[E_X|E_Y]\cdot[F] = 0 \qquad (3.4a)$$

or

$$E_X F_X + E_Y F_Y = 0 \qquad (3.4b)$$

Multiplying both sides of (3.4b) on the left hand by E_X^{-1} and on the right hand by F_Y^{-1} the last equality can be rewritten as:

$$E_X^{-1} E_Y = -F_X F_Y^{-1} \qquad (3.5)$$

Putting on the left side in place of E_X and E_Y values from (3.3a) and (3.3b) finally one obtains:

$$T^{-1}\Phi\cdot T = -F_X F_Y^{-1} \qquad (3.6)$$

The searched values of time delays as the diagonal entries of the matrix Φ are equal, according to formula (3.1b), the diagonal entries of the right-hand side matrix of (3.6).

The matrix F spans the null-subspace of space spread on vectors $[E_X|E_Y]$. Appending a non-triviality constraint FF = I to eliminate the zero solution and applying standard Lagrange techniques leads to a solution for F given by the eigenvectors corresponding to the d smallest eigenvalues of [7]:

$$[E_X|E_Y]^H [E_X|E_Y] = E\Lambda E \qquad (3.7)$$

with 2M x 2M matrix E:

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \qquad (3.8)$$

$$F = - E_{12}\cdot E_{22}^{-1} \qquad (3.9)$$

The eigendecomposition of matrix (3.9) with diagonal values Φ_{ii}

$$\Phi_{ii} = \exp(-j2\pi\cdot K\cdot p\ \tau_i/Tu) \qquad (3.10a)$$

gives the expected values of $\{\tau_i\}$:

$$\tau_i = \log(\Phi_{ii})(jTu)/ (2\pi\cdot K\cdot p) \qquad (3.10)$$

2.3.1. The ESPRIT algorithm in case of linear arrays: the proposed smoothing preprocedure

The first steps of the ESPRIT algorithm are the same as in the MUSIC and so in the case of reflected paths or SFN signals which are fully coherent it is necessary to carry out the smoothing preprocessing procedure [12]-[14]. The smoothing procedure includes the following steps (see Fig. 6):

1. In every of the two shifted arrays X and Y appoint the M-dim subarray translated by Δ taking into account condition M > d, where d is the maximal number of signal paths in the OFDM channel.

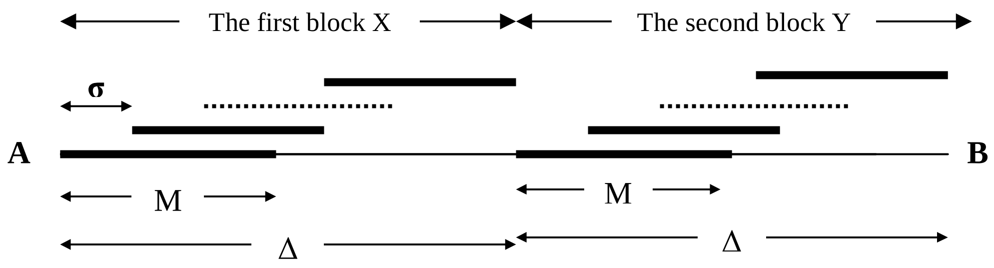


Fig. 6. Division of linear array AB with two translated by Δ blocks X and Y **into subarrays M(1), …, M(K-M+1)**

2. Let in the first array be the steering matrix A_M and noise vector N_M. The adequate steering matrix in the second array will be $A \cdot \Phi$ with diagonal matrix Φ. In case of linear transform in OFDM system the entries of rotation matrix are equal:

$$\boldsymbol{\Phi}_{(kl)} = \delta_{kl} \cdot \exp(-j2\pi\Delta \cdot \tau_i / Tu) \qquad (3.10)$$

   Here Δ – number of translated subcarriers; τ_i - time delay of the i'th path; Tu – useful part of the OFDM symbol. The vector noise is an independent one N'_M.

3. Unite the signals from both subarrays into a 2M-dim vector

$$R_M(1) = [R_1, R_2, \ldots, R_M, R_{(M+1)}, \ldots, R_{2M}]^H$$

4. The steering vector of $R_M(1)$ is equal:

$$\overline{A} = \begin{bmatrix} A \\ A \cdot \Phi \end{bmatrix} \qquad (3.11)$$

5. Calculate the covariance matrix:

$$\mathbf{R}M(1) = R_M(1)\cdot R_M(1)^H$$

6. Repeat the steps 1 – 3 for K-M+1 subarrays created from the first one by subsequent shifting by translation step σ

   The rotation matrix for consecutive translated steering matrix in smoothing processing has entries:

$$\mathbf{I}_{\boldsymbol{\sigma}\,(kl)} = \delta_{kl}\cdot\exp(-j2\pi\cdot\sigma\cdot\tau_i/\ Tu) \qquad (3.12)$$

7. The covariance matrix of the first unite vectors has the form:

$$RM(1) = \bar{A}R_S\bar{A}^H + \bar{A}N_1^H + N_1\bar{A}^H + NN^H \qquad /3.13/$$

   the second

$$RM(2) = \bar{A}I_\sigma R_S I_\sigma^H\bar{A}^H + \bar{A}I_\sigma N_2^H + N_2 I_\sigma^H\bar{A}^H + NN^H \qquad /3.13a/$$

   and finally the smoothed covariance matrix as the sum of all matrices of shifted vectors:

$$\mathbf{RM} = \bar{A}(1 + I_\sigma + I_\sigma^2 + \ldots + I_\sigma^{\Delta-M+1})R_S(1 + I_\sigma + I_\sigma^2 + \ldots + I_\sigma^{\Delta-M+1})^H\bar{A}^H + \sigma^2\cdot I_{2M} \qquad /3.14/$$

8. From the eigenvalue decomposition of the matrix **RM**:

$$\mathbf{RM}f_i = \lambda_i f_i \qquad /3.15/$$

   or

$$\bar{A}(1 + I_{\sigma f} + I_\sigma^2 + \ldots + I_\sigma^{\Delta-M+1})R_S(1 + I_\sigma + I_\sigma^2 + \ldots + I_\sigma^{\Delta-M+1})^H\,\bar{A}^H f_i = (\lambda_i - \sigma^2)\,f_i$$

   it follows that

$$\bar{A}^H f_i = 0 \qquad \text{if} \qquad \lambda_i = \sigma^2$$

   i.e. that steering vectors of matrix Ā are orthogonal to the noise subspace spanned on the noise vectors of the smoothed matrix **RM**

9. The signal vectors of the matrix RM are orthogonal to the noise subspace

The points (8) and (9) indicate, that ESPRIT processing applied to vector R_M with the steering matrix Ā composed of the steering matrixes A_M and $A_M\Phi$ goes further as in classic ESPRIT algorithm with Ā replacing A.

Presented outline of the ESPRIT theory leads to the following steps of the algorithm:

2.3.2. The main steps of the ESPRIT algorithm applied to the OFDM subcarriers:

1. Calculate estimation of the covariance matrix R from the OFDM symbol modulation factors $\{R_k\}$, k = 1,…, 2K
2. Proceed preprocessing for joint subarrays of dimension M > d within block X and Y
3. Compute the eigendecomposition of matrix (2M x 2M):
4.

$$R_M = \frac{1}{(K-M+1)} \sum_{k=1}^{K-M+1} \begin{bmatrix} R_{1M}(k) \\ R_{2M}(k) \end{bmatrix} \cdot \begin{bmatrix} R_{1M}(k) \\ R_{2M}(k) \end{bmatrix}^H \qquad /3.16/$$

5. Estimate the number of sources d, (vectors of noise-space E_N), vectors of signal space E_S
6. Decompose signal subspace vectors E_S (2M x d) into E_X (M x d) and E_Y (M x d):

$$E_S = \begin{bmatrix} E_X \\ E_Y \end{bmatrix} \qquad /3.17/$$

7. Compute the eigendecomposition of matrices (2d x M)(M x 2d) = (2d x 2d)

$$[E_x|E_y]^H \cdot [E_x|E_y] = E\Lambda E, \quad \Lambda = \mathrm{diag}(\lambda_1 > \lambda_2 > \ldots > \lambda_{2d})$$

8. Partition E into d x d submatrices:

$$E = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix}; \qquad /3.18/$$

9. Calculate the eigenvalues $\{\varphi_k\}$ of matrix F:

$$F = -E_{12}\,(E_{22})^{-1} \text{ for } k = 1, 2,\ldots,d$$

10. Estimate time delays:

$$\tau_k = j \cdot \ln(\Phi_{kk})\, T_U/(2\pi \cdot p \cdot \Delta), \qquad \text{for } k = 1,2,\ldots,d$$

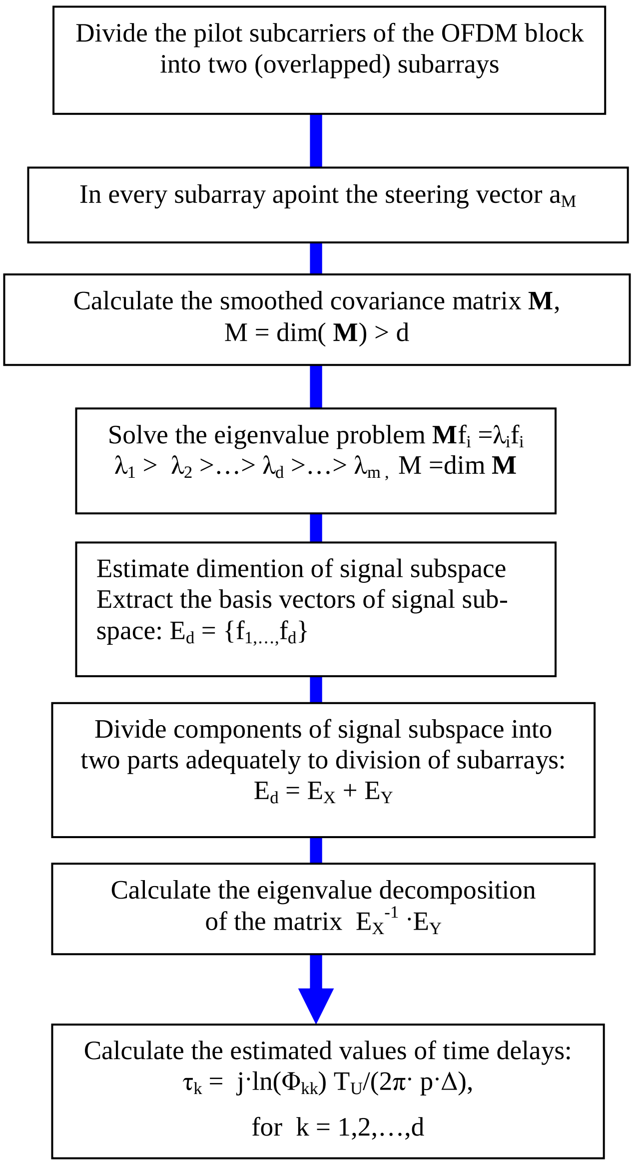


Fig. 7. The main steps of the ESPRIT algorithm

3. Estimation of the Effectiveness of the Algorithms

The effectiveness of Root-MUSIC and ESPRIT algorithms to the applications in the OFDM area are compared in this work with the help of computer simulations for selected parameters of the OFDM system, the OFDM channel and the algorithms.

The OFDM signal in wireless channel is described by the multipath delay profile (MDP) with amplitudes, time delays and demodulation angles for every path [6]. The assumed MDP profile allows constructing the pilot transmittance factors of the OFDM symbols which next, after adding the complex noise components, serve as introductory data for the algorithm processing.

The applied simulations parameters range through:

- data of the MDP profiles,
- parameters of OFDM symbols
- parameters of the algorithms.

Presented results give an overview of resolution ability of analyzed algorithms applied to the time delay estimation in the OFDM channels.

3.1. Simulation Algorithm

As shown in Section 1.1, the algorithms applied to the pilots of the OFDM symbols theoretically should allow estimating time delays of individual paths of the signal.

Parameters related with the symbols of the OFDM signal are pointed out in **Fig. 8.**

The glossary of applied terms include:

d - number of assumed paths in the MDP profile

B - frequency block of the OFDM signal

N - number of all subcarriers in OFDM symbol

Tu - useful part of the OFDM symbol

p - inter-pilot spacing in the OFDM symbol (in subcarrier units)

N/p – number of pilots in OFDM symbol

Np - number of pilot subcarriers of the OFDM symbol applied in the algorithm

M - dimension of the smoothed covariance matrix

σ - the variance of noise coefficients in all pilot subcarriers

R - vector of pilot factors in the OFDM symbol

Z - vector of noise coefficients in pilot subcarriers

RS - vector of noisy pilot factors (RS=R+Z)

The general steps of the simulation algorithm are presented in **Fig. 9**. It consists of two main parts:

3.1.1. Construction of pilot subcarriers for assumed MDP profile completed with demodulation angles.

Propagation conditions of the channel are described by the actual multipath delay profile (MDP) supplemented by 'demodulation angles' [6]. Profile MDP allows next to construct pilot subcarriers for assumed parameters of the OFDM symbols (Tu, N, p):

$$R_k = \sum_{i=1}^{d} A_i e^{-j2\pi \cdot p(k-1)\tau_i/Tu} = \sum_{i=1}^{d} A_i e^{-j2\pi(k-1)\tau_i/(Tu/p)} \qquad /3.1/$$

where:

- A_i is a set of complex amplitudes for i =1,…,d,
- Tu/p points out the limit of recognizable time delays
- index *k* enumerates the pilot factors for k = 1, 2, …, Np.

Complex DFT noise components N_k in pilot subcarriers (*1 - Np)* are realized as 'normal' real noise distribution with its Hilbert transform as its imaginary part. The noise is added to the pilot subcarriers of the OFDM symbol. The noise level is parameterized by value of the multiplier 'sigma'.

3.1.2. Application of algorithm proceeded by smoothing preprocessing.

The procedures of this part were discussed in Section 2 of this work.

An arrow on right hand side of **Figure 9** presents the back loop for the choice of the new value of chosen parameter.

The final results of estimations are exposed on single figure with assumed (x) and estimated values (o) of time delays. The data obtained for different values of analyzed parameter are presented in different levels of the figure.

3.2. Simulation Parameters

The resolution ability of algorithms depends on three groups of parameters:

1. The OFDM channel parameters of the Multipath Delay Profile completed with demodulating angles:
   - Number of paths (d)
   - Amplitudes, time delays, phases of individual paths
   - Minimal time delay difference between subsequent paths ($\Delta\tau$)
   - Relation between amplitudes of the neighboring paths
   - Ratio of the subcarrier amplitudes and adequate noise coefficients

2. The OFDM signal is described by parameters:
   - Number of all subcarriers (N)
   - Useful part of the OFDM symbol (Tu)
   - Time resolution of the system (1/B)
   - Inter-pilot spacing (p) in OFDM symbol
   - Number of all pilot subcarriers (N/p) in OFDM symbol

3. The algorithm applied to OFDM signal depends on:
   - Dimension of the smoothing parameter M [6]
   - Number of pilots Np used in the smoothing preprocessing

The smoothing factor L = Np-M+1 depends on last two parameters.

Presented results of the simulations express dependence of algorithm resolution on selected proposed parameters:

Np	- portion of symbol pilot subcarriers applied in algorithm
σ	- proportionality factor of the noise DFT coefficients distributed in pilot subcarriers
M	- dimension of the smoothing parameter in preprocessing
$\Delta\tau$	- time delay difference between pairs of subsequent paths in MDP profile
p	- inter-pilot spacing in OFDM symbol

In should be noticed that not all subcarriers need to be applied in the algorithm. Parameter Np describes the part of all pilot subcarriers that are applied in algorithm. It limits the smoothing parameter – number of smoothing matrices included in averaging. The algorithm dependence on this parameter is presented in point 4.1. Number N/p of all pilot subcarriers in OFDM symbol in relation to the number of all subcarriers appoint the limit of estimated time delays [6] as demonstrated in point 4.5.

The results of algorithm simulations depend on ratio of MDP amplitudes and adequate DFT noise coefficients on pilots. In point 4.2, where this dependence is demonstrated, the amplitudes, as well as noise coefficients, are expressed in linear scale.

The smoothing preprocessing demands that parameter M should have been greater then number of paths. In case of unknown number of paths in OFDM channel it is of importance to know influence of relation between number of paths and dimension of smoothing matrix M on algorithm resolution. This problem is analyzed in point 4.3.

The theoretical time resolution of OFDM channel is limited by 1/B = Tu/N. How practically MUSIC resolution depends on time delay differences between MDP paths. It is analyzed in point 4.4 for appointed simulation conditions.

3.3. The necessary conditions for time delay estimation of the OFDM paths

In broadband digital systems [1- 4] the channel estimation is supported by pilot subcarriers. Its distribution is dictated by the aim of signal demodulation and not for the paths parameter estimation. As pointed out in [6] there are two conditions for unanimous application of the MUSIC algorithm to pilots in the current OFDM symbol:

a) $p \leq Tu/Tg$ i.e. inter-pilot separation cannot overcome value Tu/ Tg, where
- Tu – is a useful (orthogonal) part of the OFDM symbol,
- Tg – is a guard interval

b) $Np \geq 3d/2$, Np – is a number of pilot subcarriers in OFDM symbol applied in algorithm

In DRM system condition (a) is not fulfilled and so it demands grouping the OFDM symbols [7]. Algorithms Root-MUSIC and ESPRIT as founded on MUSIC algorithm also demand fulfilling conditions (a) and (b).

c) Exclusively in the ESPRIT algorithm the number of the applied pilots in the OFDM symbol is equal twice the number of pilots applied in MUSIC algorithm:

$$2(M+L-1) > 2(3d/2+L-1)$$

4. Simulation results

The conditions of the simulations for chosen simulation parameters are shortly presented in following sections.

4.1. Resolution of time delays as a function of the number of pilot subcarriers (Np) applied in the algorithm

Number Np of pilots applied in MUSIC algorithm is independent of all N/p pilots in OFDM symbol. Value of Np should be greater then dimension of the smoothed matrix M and not less then 3d/2 as pointed in Sec. 3.3, point b. In order to check what happens if this condition is not fulfilled the range of parameter limit in simulations passes this value.

Construction of the smoothed matrix M needs not to involve all pilot subcarriers Np of the OFDM symbol. In order to simplify calculations for M construction only part of pilots could be used to this aim. In this point it is assumed that frequency block B is constant so that reception conditions of the multipath signal do not change. In order to observe relation between number of pilot subcarrierrs involved in construction of the smoothed matrix M and resolution of the MUSIC algorithm the $Np \leq N/p$ is treated as parameter growing smaller from 48 to 24. The noise distribution in pilot subcarriers remains constant in all simulations.

The influence of the number of pilot subcarriers Np on evaluation process of time delay of MUSIC algorithm is demonstrated for:

- Multipath delay profile MDP presented in Fig. 10 contains d = 20 paths. It is assumed that all path appear up to Tu/p. It allows avoiding the complications resulting 'ghost' paths, as shown in point 4.5.
- Useful part of the OFDM symbol Tu = 1
- The number of all subcarriers in an OFDM symbol is constant and equal N =192
- Pilot inter-carrier spacing p is assumed constant for every Np and equal p = 4
- The smoothing matrix dimension M = dim(**M**) is set M = 22
- Resolution of the channel $\Delta t = 1/B = Tu/N = 0{,}0052$
- Distribution of DFT noise coefficients is set for upper limit of Np and remains unchanged when passing to smaller Np. Variation of the noise coefficients sigma is set $\sigma = 5$ and 10 for every Np

Assumed limits of parameter:

- The value of Np is changed within limits Np = 24:4:48

The values depending on assumed parameters:

- The resulting smoothed covariance matrix **M** depends on Np through the smoothing factor L= Np –M+1 = 2:4:26

The time delays in MDP profile are expressed as fractions of the Tu / p= 0,25. All paths have delays below Tu/p and condition (a) of 3.1 is fulfilled. The condition (b) for assumed d = 20 paths in MDP profile is fulfilled only for Np > 3d/2 = 30.

Results of simulations presented in **Figures 11A – 12B** for σ = 5 or 10 indicate that:

- For Np ≤ 30 the number of estimated values is distinctly smaller then number of paths.
- For Np over 32 but less then 48 the high value of noise factor (σ = 10) do not allow to estimate the time delay of the last smallest path.
- Only for Np over 48 the high smoothing factor leads to mitigating the influence of noise components allowing extracting the highest value of time delay.

4.2. Resolution of time delays as a function of the noise multiplier (σ)

The influence of noise coefficients in pilot subcarriers on time delay evaluation process is presented through dependence on proportionality factor σ. Each of assumed noise coefficients distributed among pilots is multiplied by common factor sigma **(σ)**. In presented simulations values of σ are changed from 1 to 15.

Simulation conditions:

- Multipath delay profile MDP the same as in **Fig. 10** with 20 paths
- Dimension of smoothing matrix M = 22
- Number of all subcarriers N = 192
- Useful part of OFDM symbol Tu = 1
- Number of pilot subcarriers in OFDM symbol Np = 32, 48
- Smoothing parameter L = 11, 27
- Pilot subcarrier spacing p = 4

Assumed limits of parameter:

- The value of σ is changed within limits σ = 1:2:15

Depending values:

- The resulting smoothed covariance matrix **M** depends on σ through the noised pilot factors /1/

The estimated values of time delays are gathered in **Figure 13 - 14** for Np equal 32 and 48. Comparison of (a) and (b) points out that with growing value of Np influence of σ decreases:

- In case of Np = 32 even for σ = 1 estimates do not fit the MDP values
- For Np = 48 up to σ = 9 results nearly cover assumed values

4.3. Resolution of time delays as a function of the dimension of the smoothed covariance matrix M

The dimension M of the smoothed covariance matrix **M** is parameter of the MUSIC algorithm. The conditions of the smoothing procedure demand M > d, where d is the number of paths in OFDM channel [6]. In practice the number of paths in signal is a priori unknown and only upper limits of *d* can be estimated from measurements in different frequency bands. It is of interest how MUSIC algorithm acts when parameter M happens also beneath this limit. All other simulation parameters are constant.

Simulation conditions

- Multipath delay profile MDP with s = 20 paths (Fig. 1)
- Tu = 1; pilot subcarrier spacing p = 4
- Number of pilot subcarriers in OFDM symbol Np = 32 or 48
- Noise factor σ = 5 or 10

Simulation parameter range:

- Dimension of the smoothed matrix M = 17:1:25 i.e. d-3:1:d+5;

Parameters influenced by value of M:

- The smoothing factor L = Np-M+1
- The matrix M eigen-decomposition and dimension of noise subspace

Results of simulations are presented in **Fig. 15 - 18** for different Np and sigma. Channel resolution in all cases is higher then minimal time differences between MDP paths.

Comparison of data leads to conclusions:

- For *M* beneath number of MDP paths *d* the number of estimated delays is smaller then number of paths
- If greater then d, value of M has little influence on estimation results
- Number *Np* of pilots included in smoothing decides about estimate fittings. The minimal value of Np for avoiding coherency (point 3.1 b) is equal 3d/2 = 30. Though both Np = 32

and 48 fulfill condition 3.1b only for Np = 48 estimates include all paths for both values of sigma.

4.4. Time delay resolution of algorithm as a function of the delay differences (Δτ) between subsequent OFDM paths

Time differences between subsequent paths in the MDP profile reflect properties of propagation channel.

The time resolution of MUSIC algorithm is demonstrated for changing time parameters of MDP profiles. In **Fig. 19**, besides the first and last paths, there appear five pairs of paths with different amplitudes. The time separation within paths in pairs is changed in twelve equal steps from maximum value 0.015Tu to zero (**Fig. 19**). Parameters of MUSIC (Np, M), signal (N, p) and noise remain constant for all MDP profiles.

Simulation conditions:

- MDP profile with 12 paths
- Dimension of smoothing matrix M = 15
- Useful part of OFDM symbol Tu = 1
- Inter-pilot spacing p = 4
- Number of all subcarriers N = 192
- The number of pilot subcarriers in OFDM symbol Np = 32and 48
- Time resolution of the channel $\Delta t = 1/B = 0.0052$
- Variance of the distribution of DFT noise coefficients $\sigma = 2$

Parameters changing with **Δτ**:

- Multipath delay profile MDP starting like in **Fig.19** and changing time delays in pairs within limits 0.06 Tu/p = 0.015Tu down to 0.

The MUSIC theoretical time resolution is limited by the receiver channel time resolution 1/B = Tu/N =0.0052Tu. Results of estimations, gathered in **Fig. 20 - 21** for Np = 32 and 48, show that:

- For assumed noise level the algorithm allows to distinguish subsequent paths for $\Delta\tau \geq 0.008$Tu.
- For time differences below 0.006Tu the pairs are treated as single paths.

- Below 0.005 there appear paths with unexpected time delay positions in between ‘lost’ paths. Creation of such paths can be analyzed by observing changes of adequate MUSIC cost functions.

4.5. Results of algorithm application in case of paths with time delays over Tu/p.

In the case of paths with time delays over the value Tu/p the MUSIC algorithm loses its estimation ability. It is a consequence of expressions for pilot subcarriers /1/.

It is assumed that OFDM frequency block B is of constant width. Resolution of the channel is thus equal 1/B = Tu/N = 1/N for Tu=1. The time delay, estimated in Tu/N units, appears only in exponentials $\exp\{-j2\pi(k-1)\tau/(Tu/p)\}$, and so is periodic with period Tu/ p.

In order to check results of simulations in case of $\tau > Tu/p$ the MDP profile with paths up to 0.85 Tu was chosen in **Fig. 22**. The different values of parameter ‘p’, for other parameters unchanged, allow to demonstrate results of MUSIC estimations for paths with $\tau > Tu/p$. The OFDM paths with time delay over Tu/p are estimated modulo Tu/p creating the ‘ghost’ values [6]. Appearance of ‘ghosts’ is exposed in Fig. for changing parameter ‘p’.

Simulation conditions:

- MDP profile presented in **Fig. 22** with d = 10 paths
- Dimension of smoothed matrix M = 20.
- Number of all sub carriers in an OFDM symbol N = 192
- Number of pilots applied in algorithm Np = 32.
- Useful part of the OFDM symbols Tu = 1
- Resolution of the channel $\Delta t = 1/B = Tu/N = 0{,}0052$
- Variation factor of noise coefficients $\sigma = 2, 5$

Parameter changing with p = 1:1:6

- Value of Tu/p

In MDP profile assumed time intervals between subsequent paths are neartly one order higher then time resolution of the OFDM channel. Because of this the value Np = 32 is enough for exact delay fittings. When inter-pilot spacing parameter p grows from 1 to 8 the estimated values of τ appear limited by (Tu/p). The values of τ over Tu/p are estimated as [τ mod(Tu/p)]. If this new values do not cover values of τ beneath Tu/p there appear additional points, ‘ghost’ ones, according to [6].

The smoothing factor L= Np –M+1=13 for all estimations. The maximal time delay interval be

The summary of MUSIC estimation results is presented in **Fig. 23** for σ = 5. Depending on inter-pilot spacing *p* the range of faithful time estimations decreases from Tu = 1 (for p = 1) to Tu/6 = 0.167 for p = 6.

5. Conclusions

Presented results of simulations exemplify the application of the algorithms MUSIC and Root-MUSIC to the time delay estimation in the OFDM channels. Simulations of algorithm ESPRIT applied to the OFDM subcarriers in case of much higher number of signal paths then three have proved that algorithm ESPRIT cannot fulfill the role of means for time delay estimation in OFDM signal. On the other hand the number of data demanded in the ESPRIT algorithm for estimation the assumed number of paths d overcomes the data number in algorithm MUSIC or Root-MUSIC. It thus appeared that usefulness of the algorithm ESPRIT for applications for OFDM signal estimation is low and so the simulation data of this algorithm are not placed in presented material.

The presented simulation results compare the data obtained in application of the MUSIC and Root-MUSIC algorithms to the time delay estimation of the OFDM paths in different simulated environments.

Depending on parameters of assumed multipath delay profile, distribution of noise coefficients in pilots, parameters of algorithms and the OFDM symbols - the resolution of the algorithm considerably differs.

Results of simulations allow drawing the general conclusions:

- Decisive influence on accuracy of algorithm has channel resolution 1/B = N/Tu. Paths with time differences beneath the time resolution (Tu/N) cannot be distinguished.
- The more pilot subcarriers (Np) is included into estimation the better is coherency dissolving between paths, noise mitigation and so higher resolution of the algorithm
- Resolution depends on time delay differences between neighboring paths in MDP profile and their amplitude relations
- In case of paths with time delays over limit Tu / p there appear additional ‘ghost’ delays
- In the case when cost function omits some paths there appear additional points of estimation having no counterpart in delayed signals. These cases demand further analysis and additional improvements of estimation algorithm.

Generally it appears that effectiveness of algorithm applied to the time delay estimation in OFDM channels depends on relations between parameters of the OFDM symbols and algorithm itself . In real applications the choice of parameters should take into account the specific conditions of definite cases.

In changing propagation conditions of DRM systems the method could be exploited as an indicator of changes. Appearance of new paths should be noticed through estimation of the new time delays.

In case of OFDM SFN networks, where only signals from nearest transmitters play role, the assumptions of the method are fulfilled and the method could serve as an on-line measure of differences between time delays of different paths.

At last it should be noticed that thanks to the analogy OFDM – DOA [6] system OFDM can serve as laboratory for DOA algorithms. Effectiveness of the DOA algorithms can be easier verified in the case of the OFDM system with single antenna and serially produced hardware.

The adaptive choice of the guard interval parameter for transmission in different localisations also demands the simple method for setting the time delays in specific regions or cells.

Presented results of algorithm applications for time delay estimation in OFDM systems take advantage of pilots of single OFDM symbol. Simulations suggest that control of time delays in OFDM signal in prospect can be estimated on-line. Such a prospect can lead to adaptive guard interval choice and so to the adaptive optimal throughput in OFDM data stream.

The work on more effective and more sensitive DOA algorithms is going on [15] – [18] in the leading signal processing world centers.

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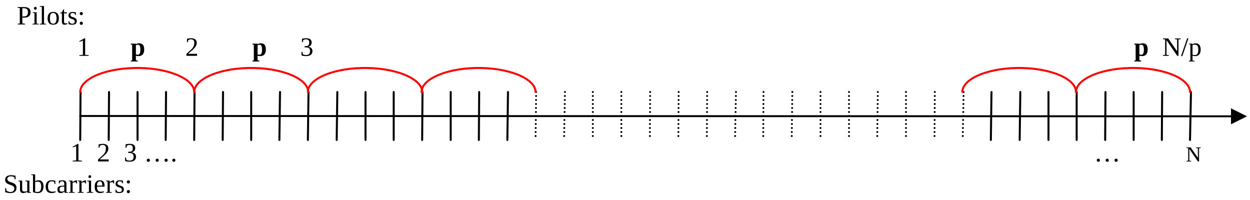

Fig. 8. Distribution of pilot subcarries in the OFDM symbol

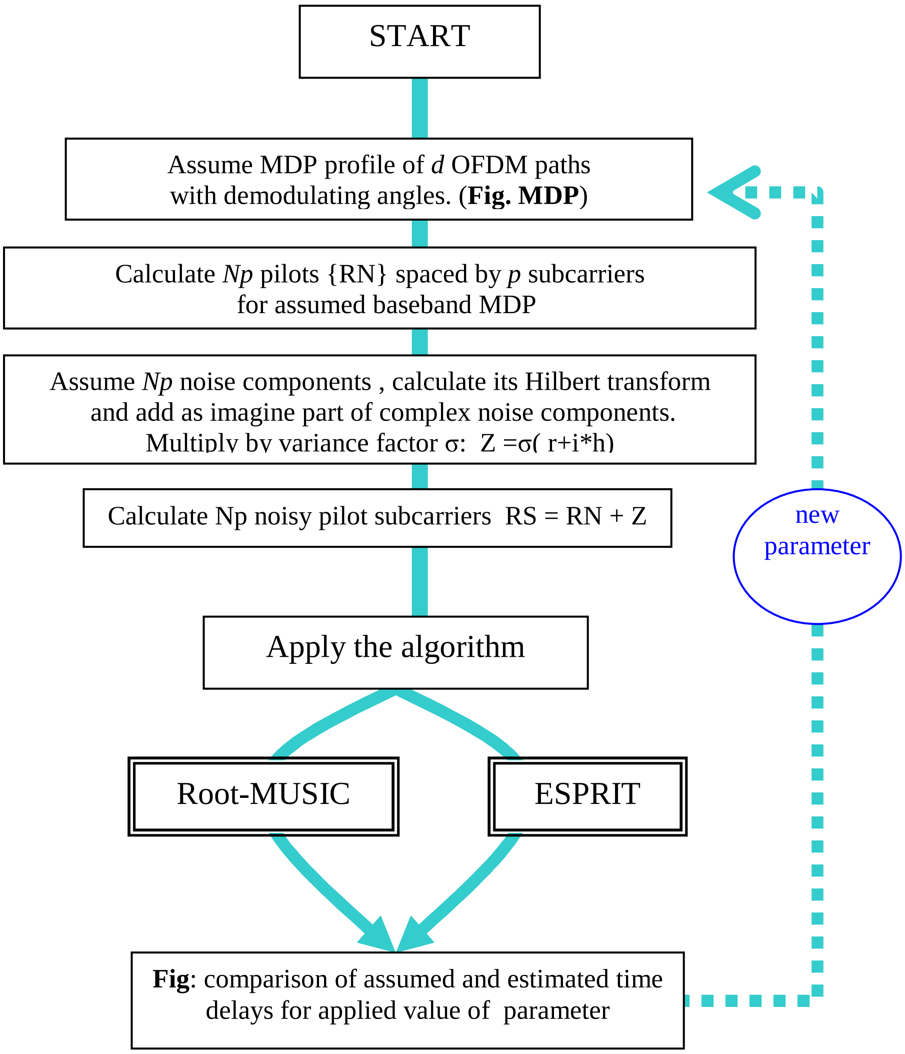

Fig. 9. Simulation procedures presenting application of R-MUSIC and ESPRIT algorithms for time delay estimation in OFDM channel

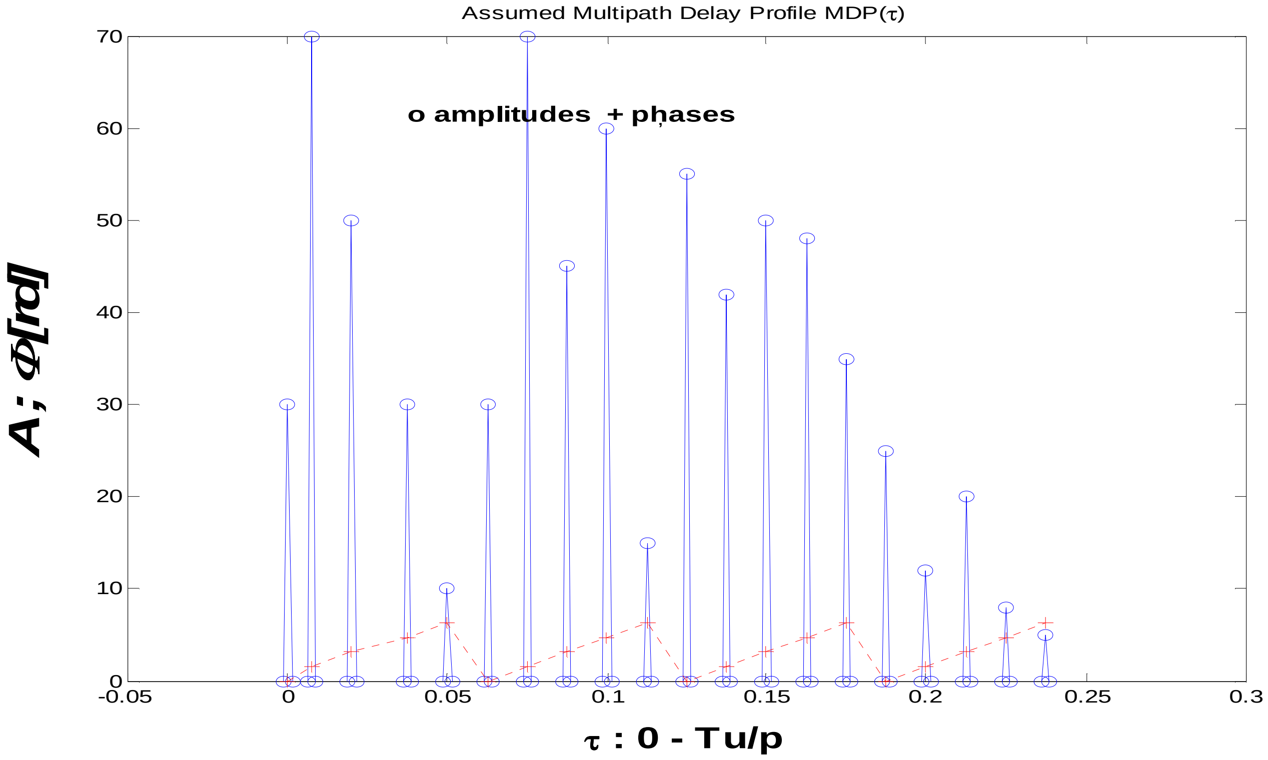

Fig. 10. The OFDM multipath delay profile for evaluation the influence of Np, sigma and M parameters on resolution of the algorithm

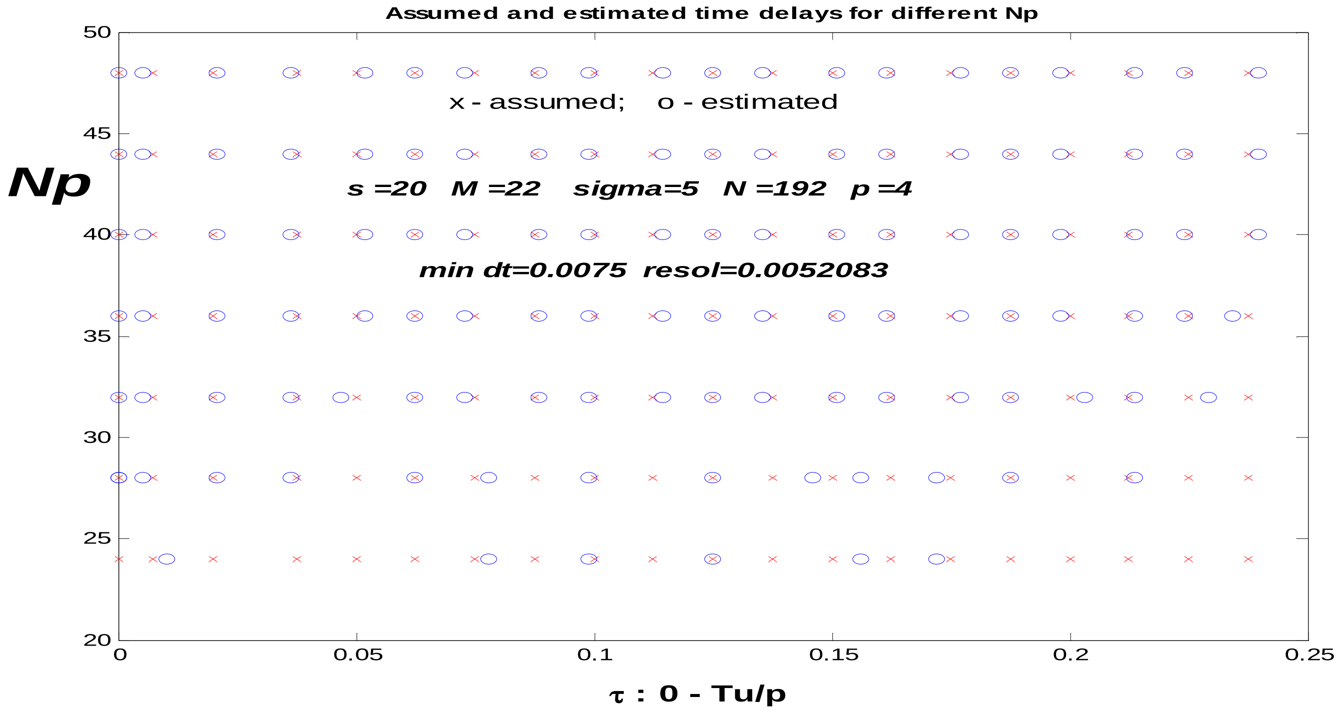

Fig. 11 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different number of the pilot subcarriers Np

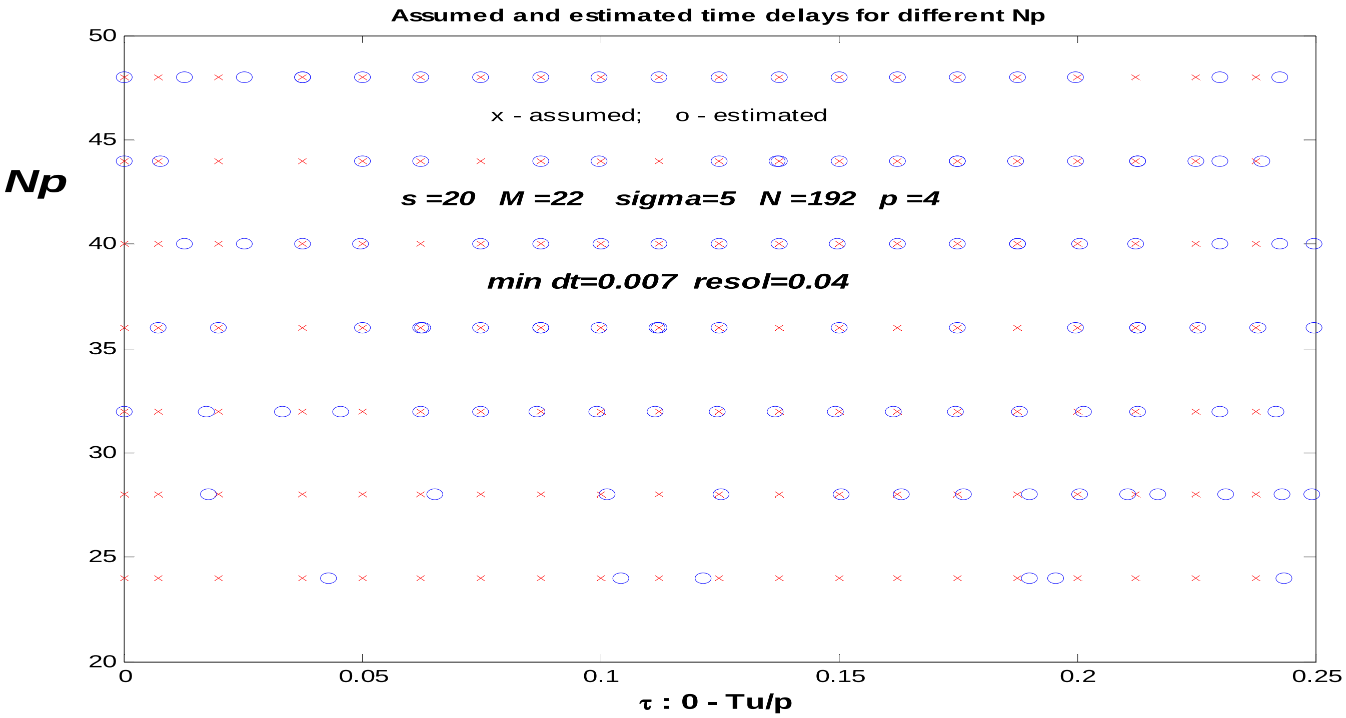

Fig. 11 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different number of the pilot subcarriers Np

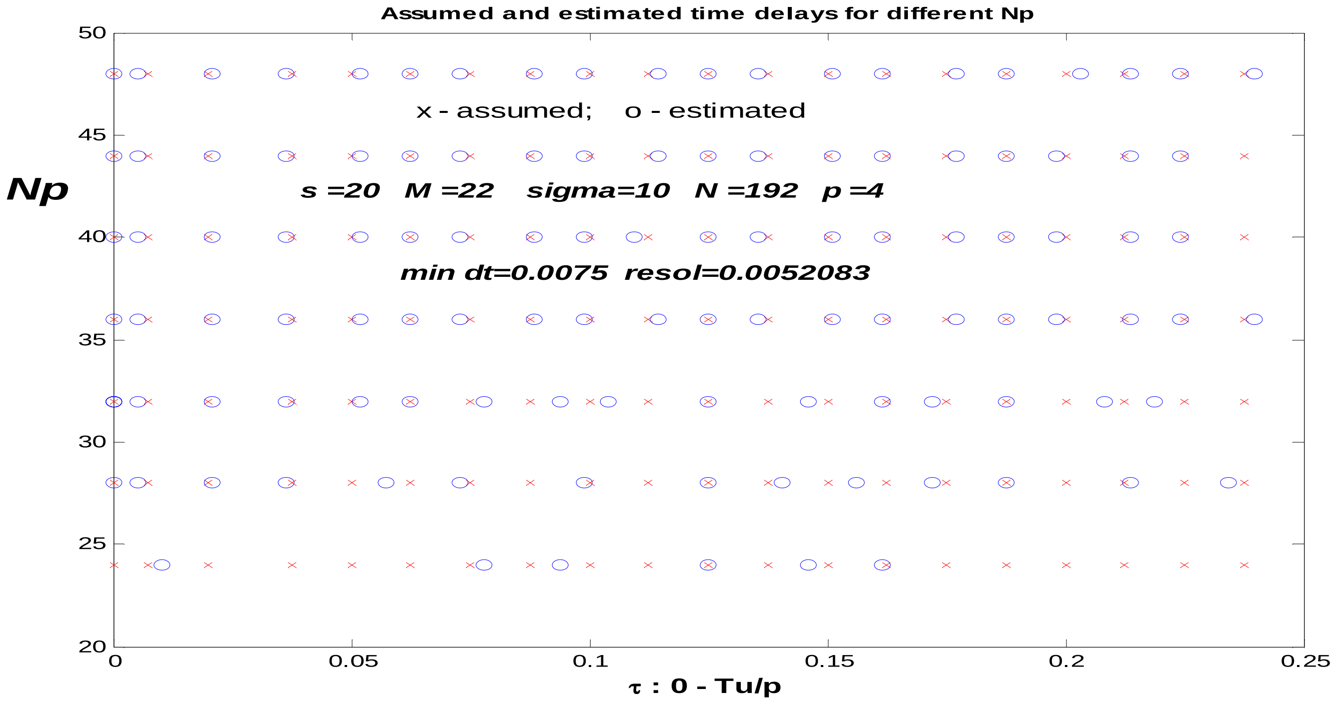

Fig. 12 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different number of the pilot subcarriers Np

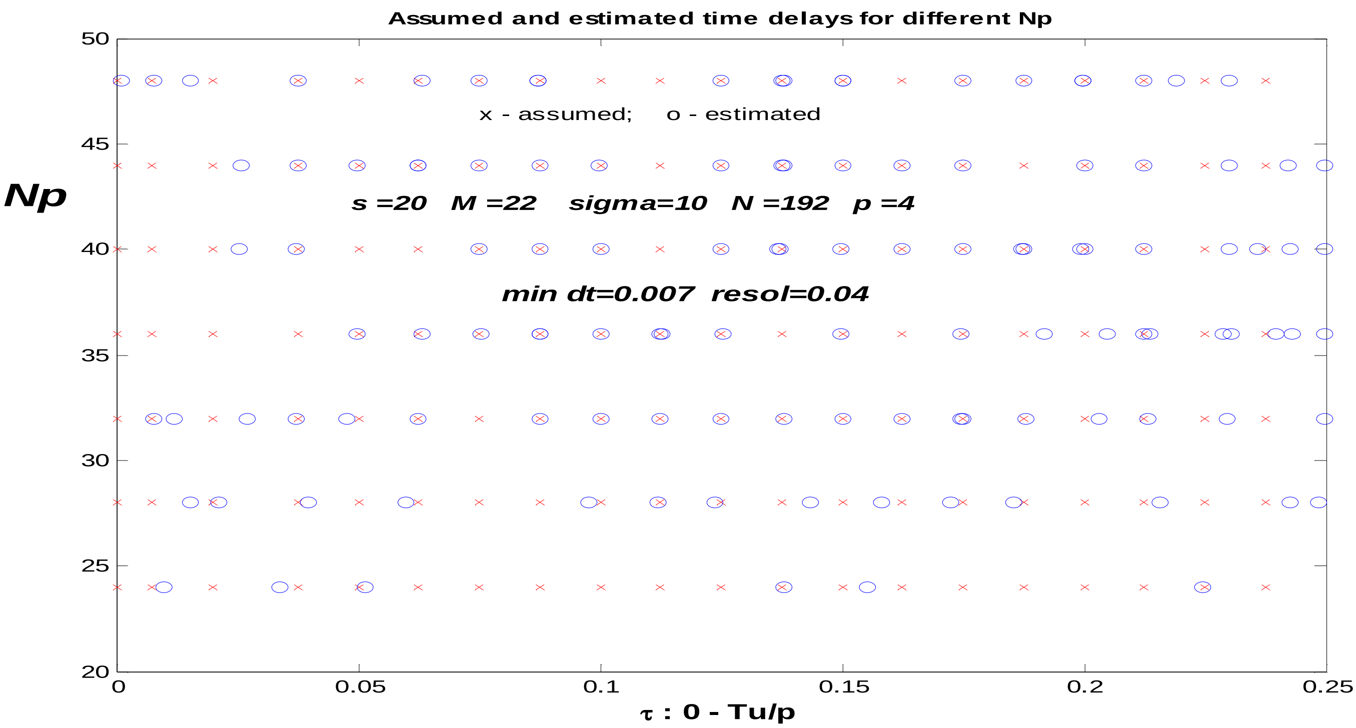

Fig.12. B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different number of the pilot subcarriers Np

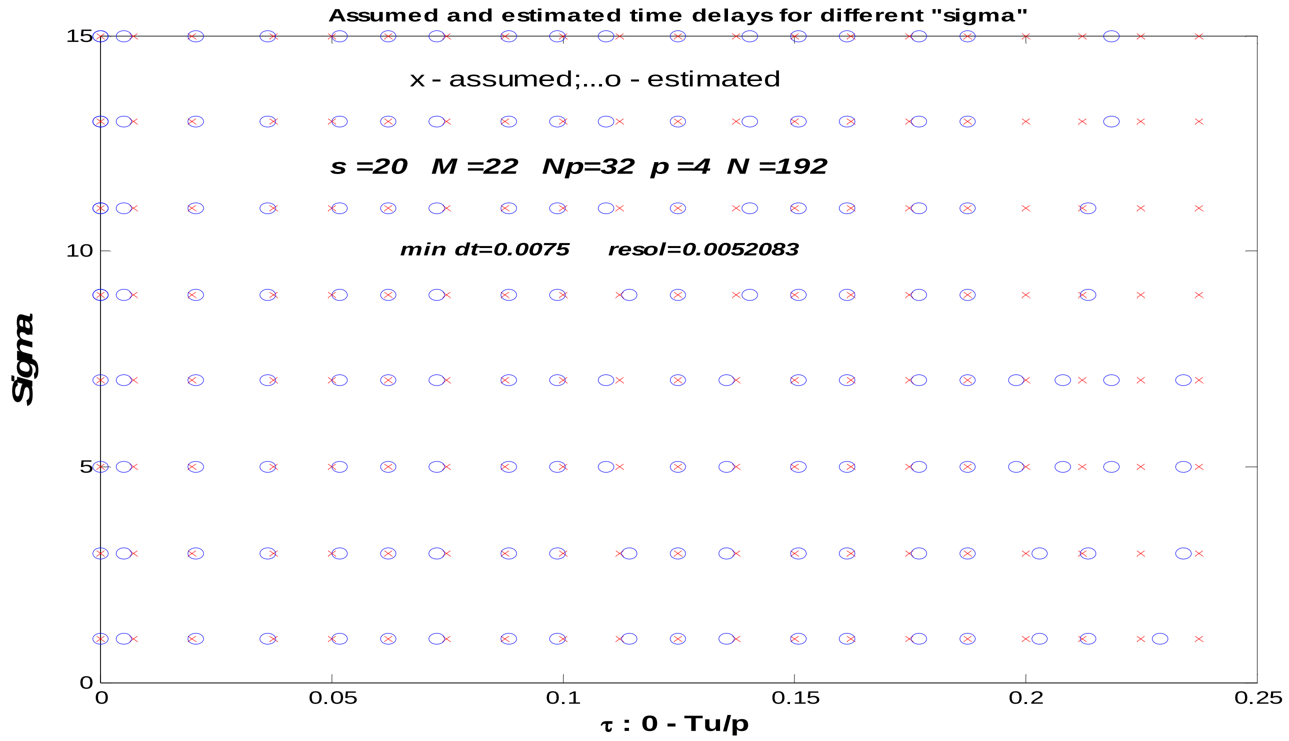

Fig. 13 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different variation of noise components sigma (σ)

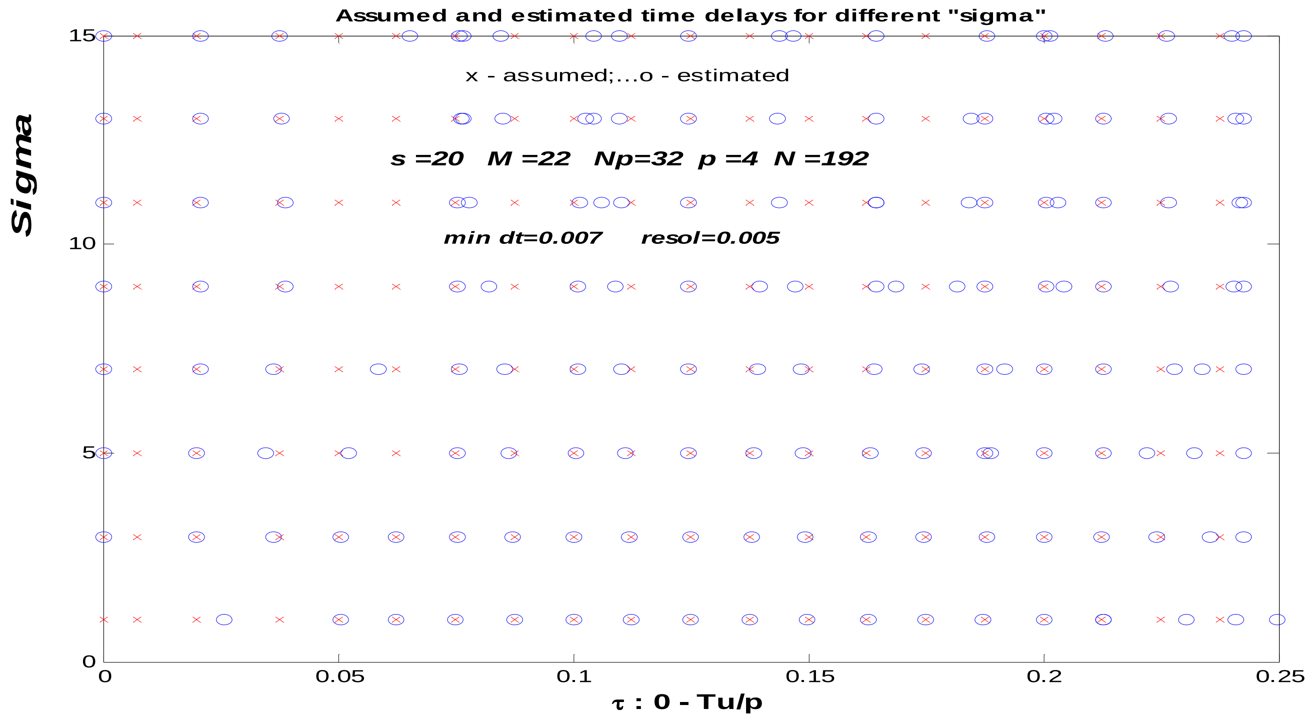

Fig. 13 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different variation of noise components sigma (σ)

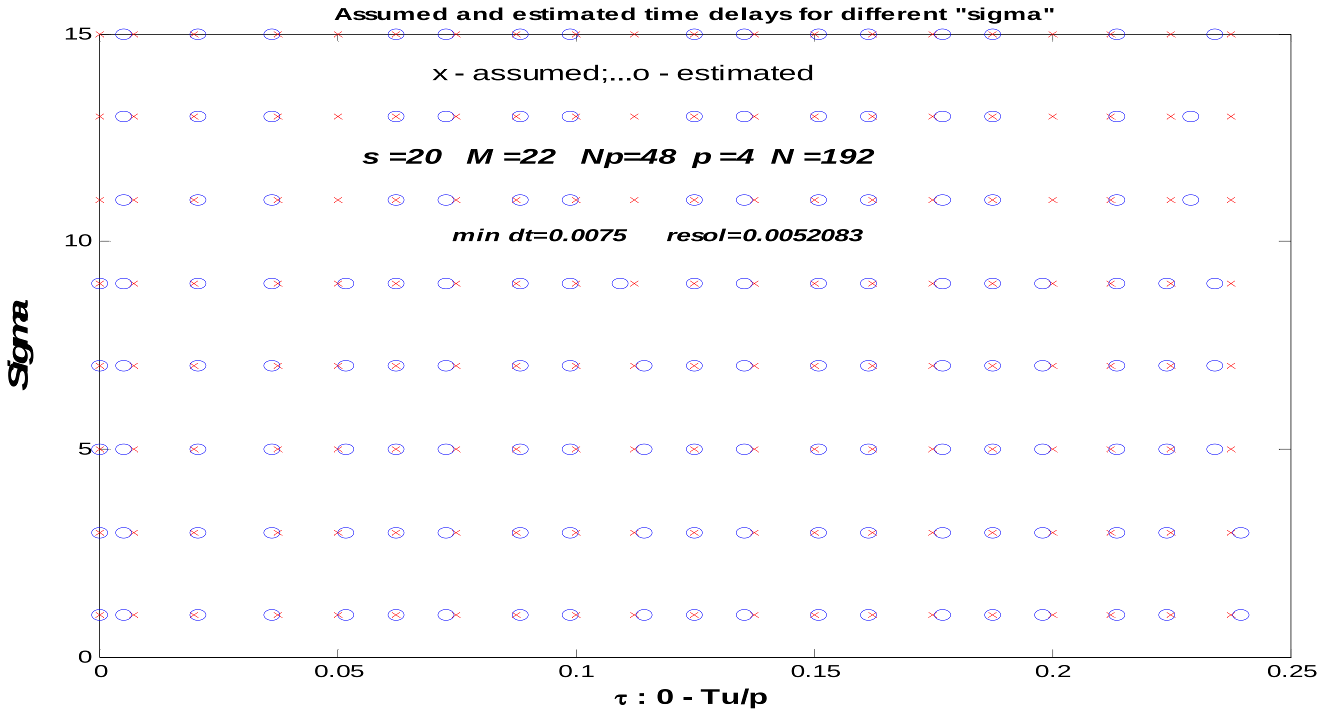

Fig. 14 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different variation of noise components sigma (σ)

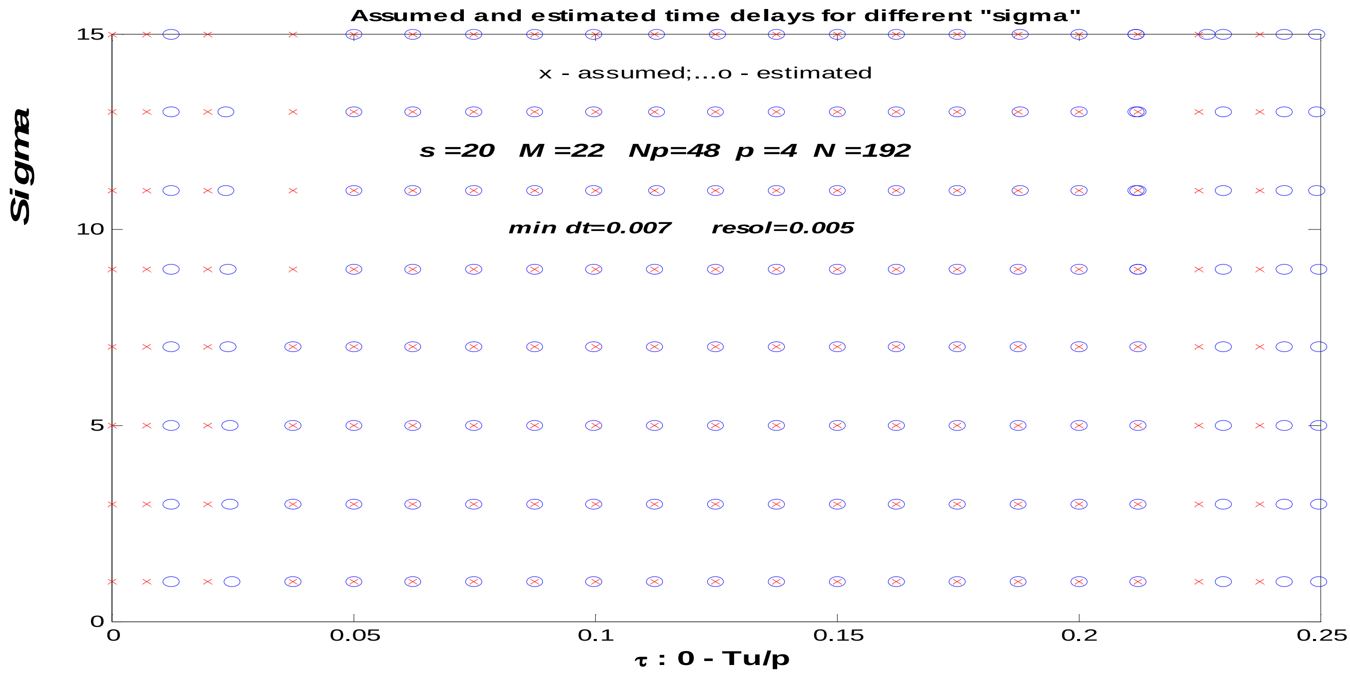

Fig. 14 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different variation of noise components sigma (σ)

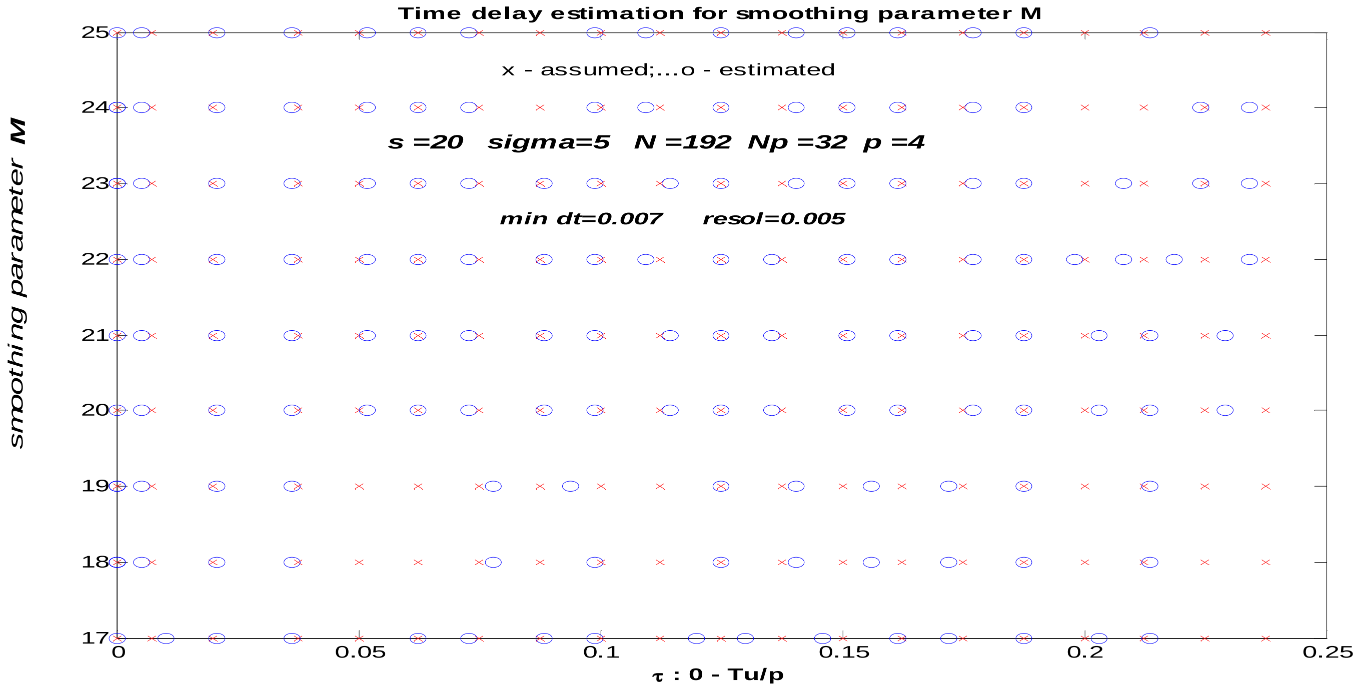

Fig.15 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

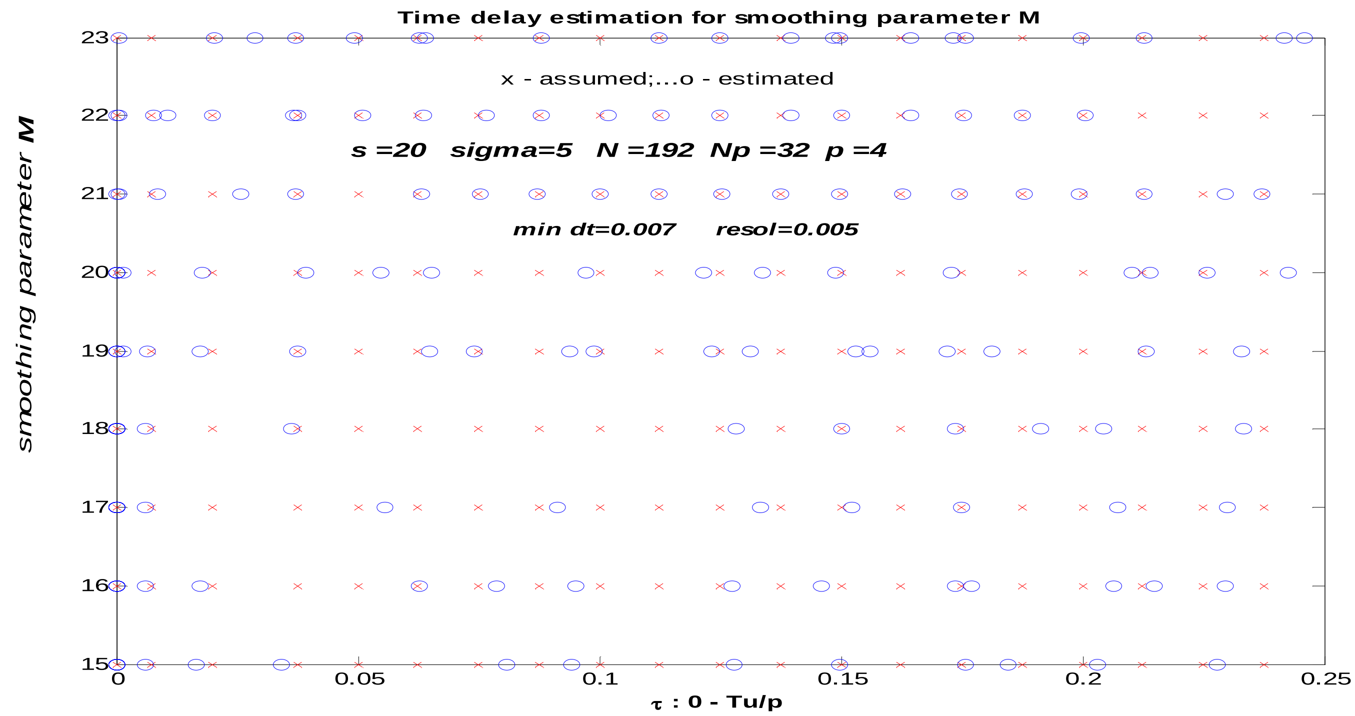

Fig. 15 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

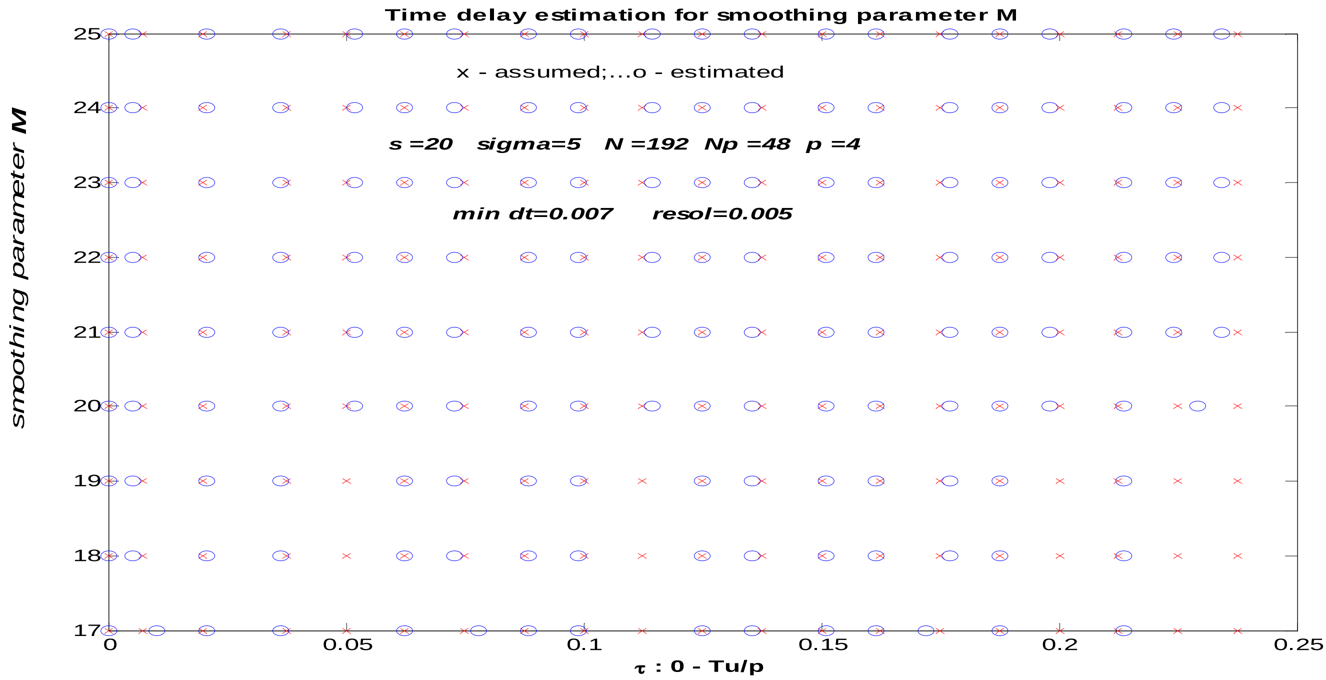

Fig. 16 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

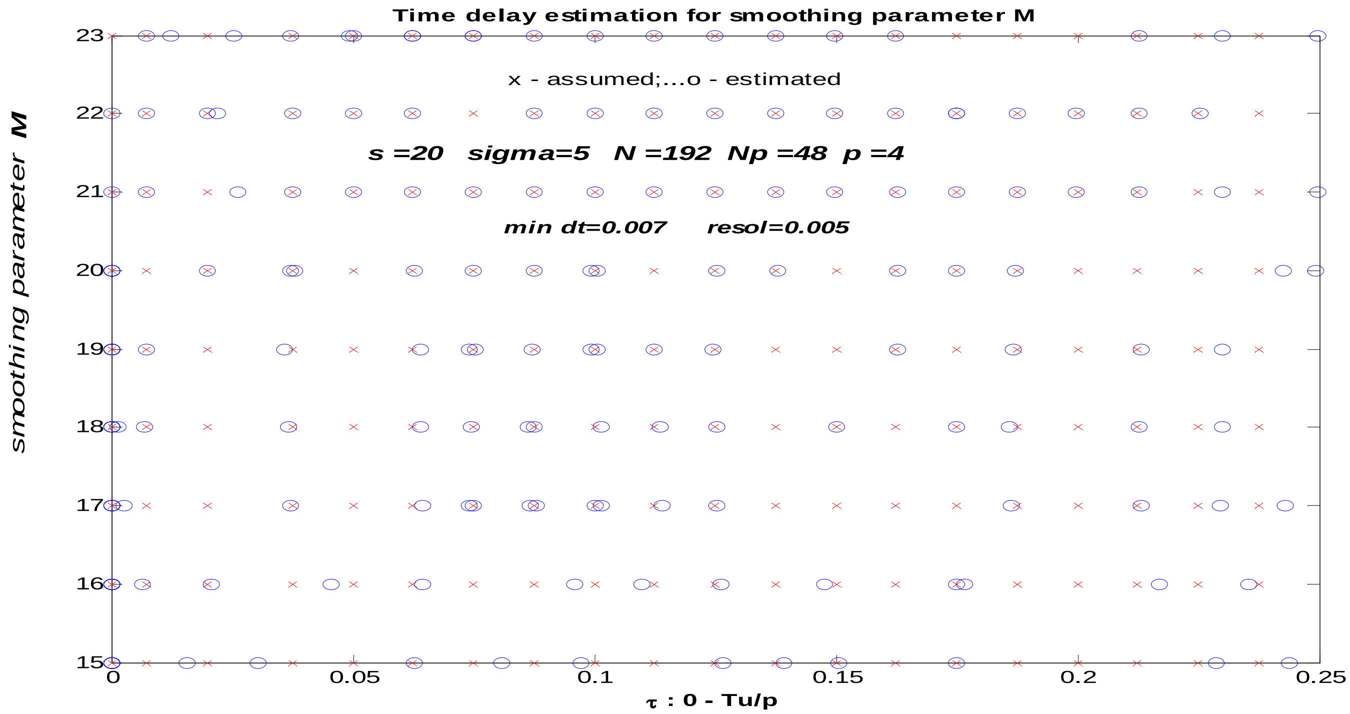

Fig. 16 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

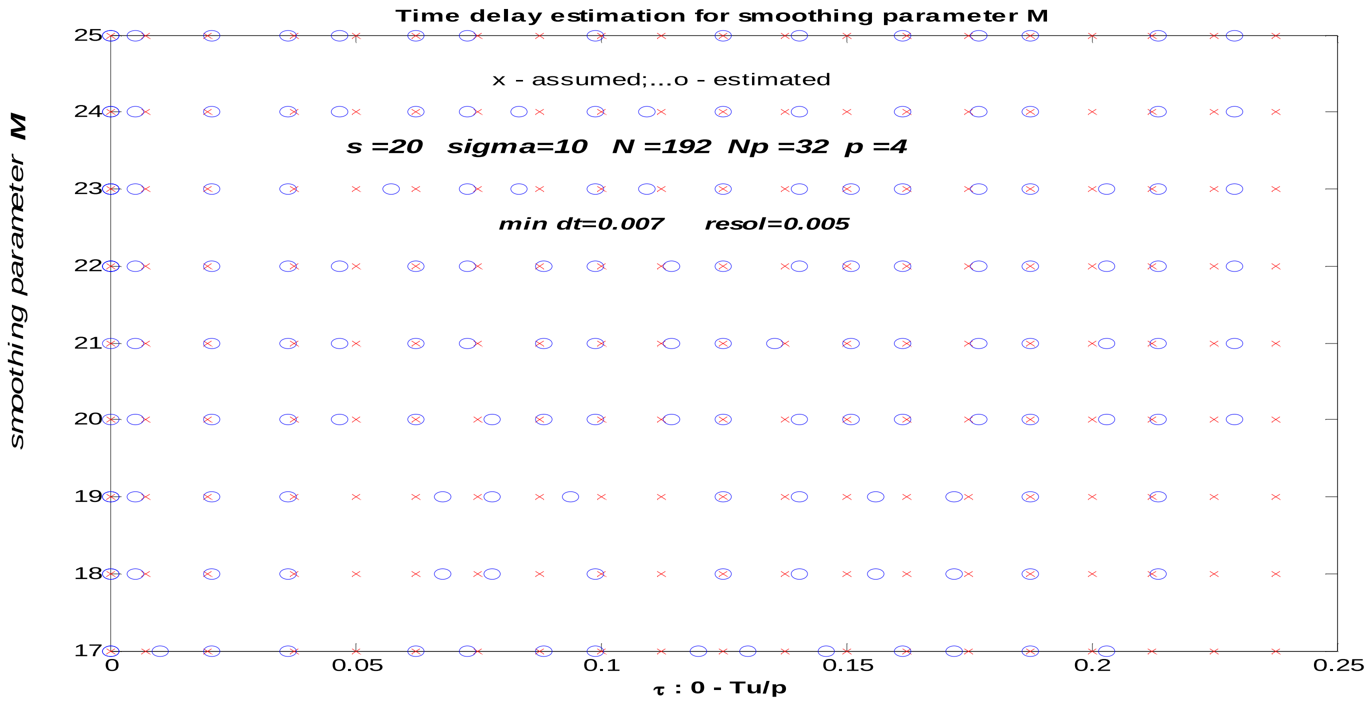

Fig. 17 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

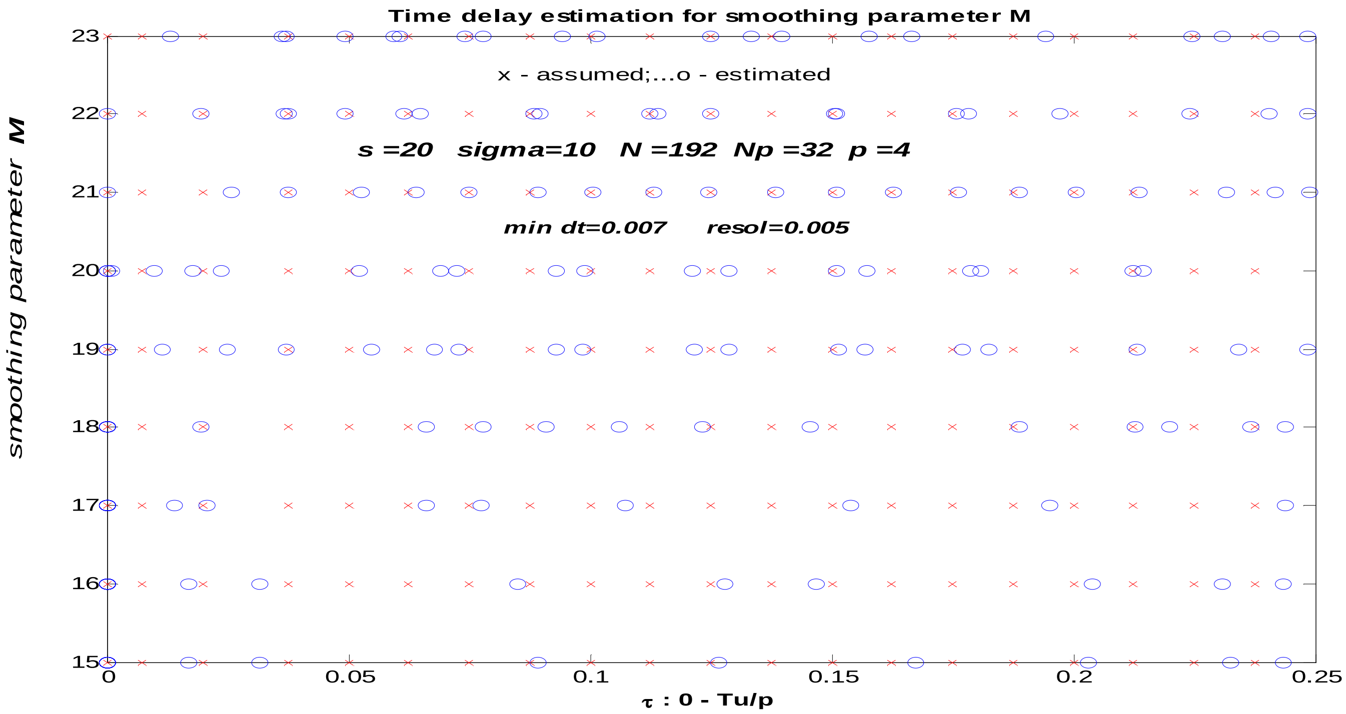

Fig. 17 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

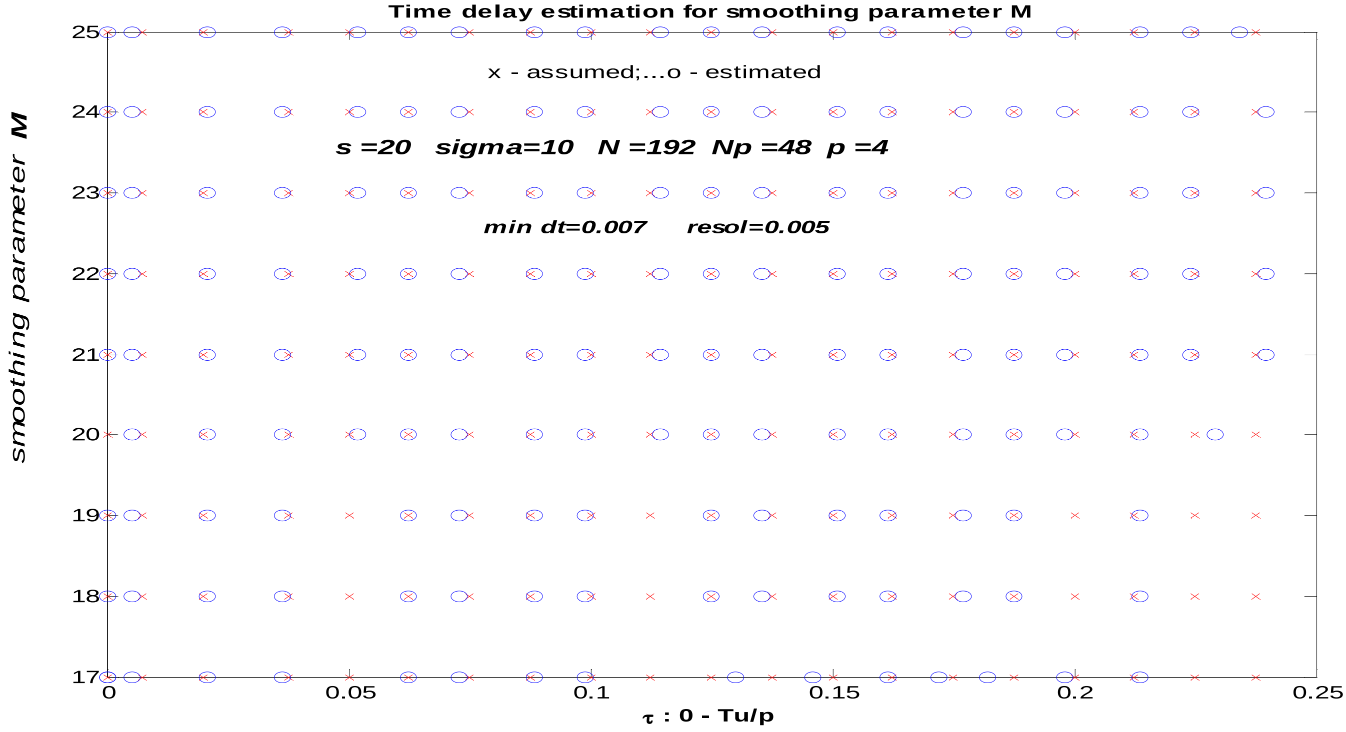

Fig. 18 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

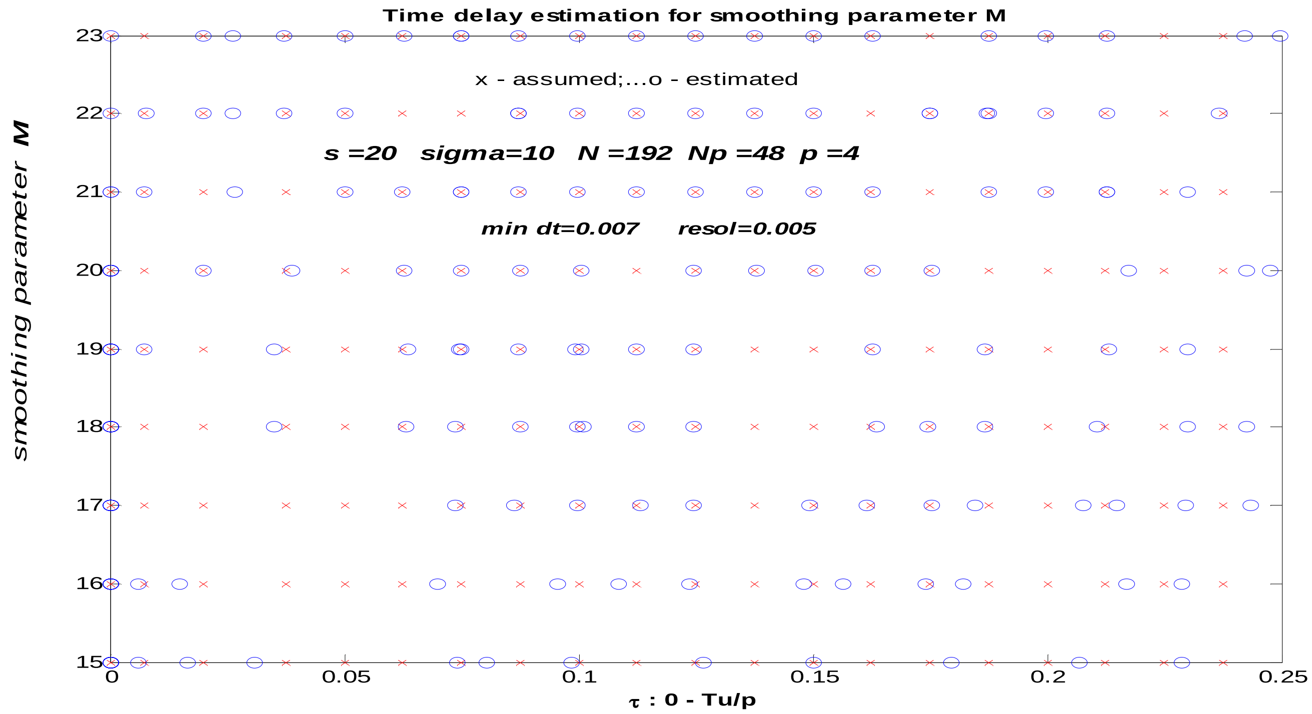

Fig. 18 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different dim M

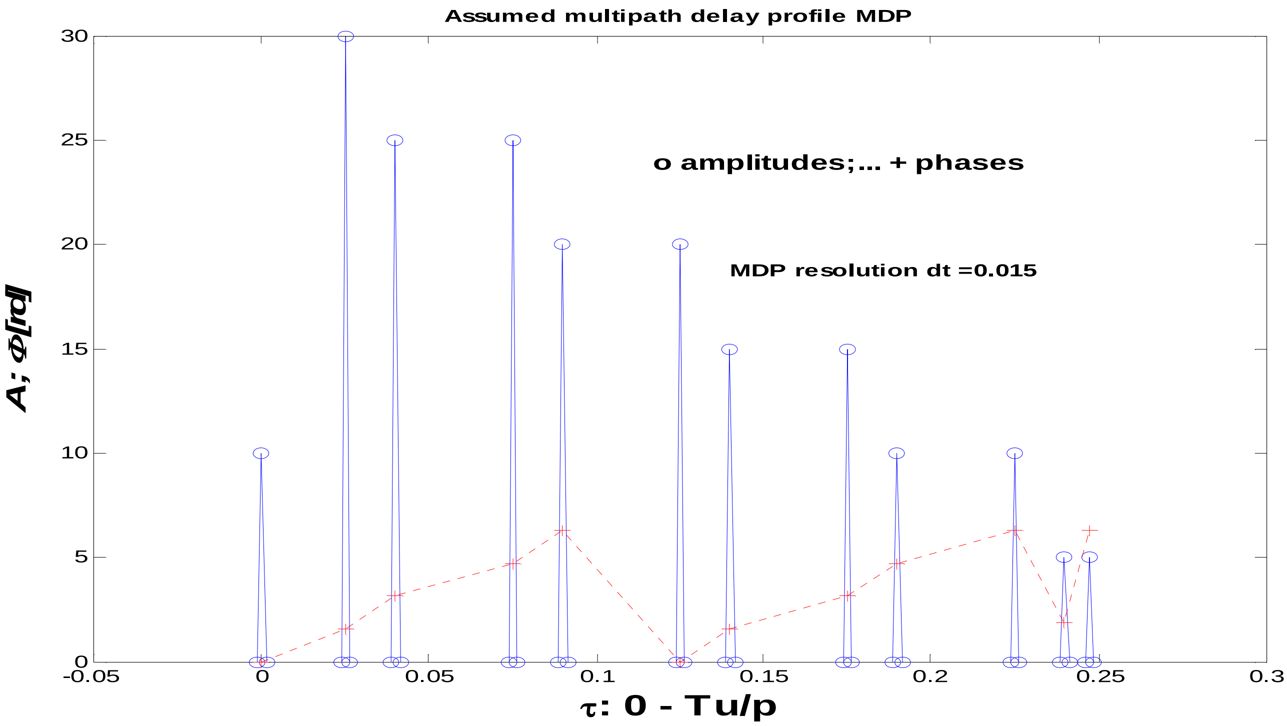

Fig. 19a. MDP profile with maximal assumed time delay differences

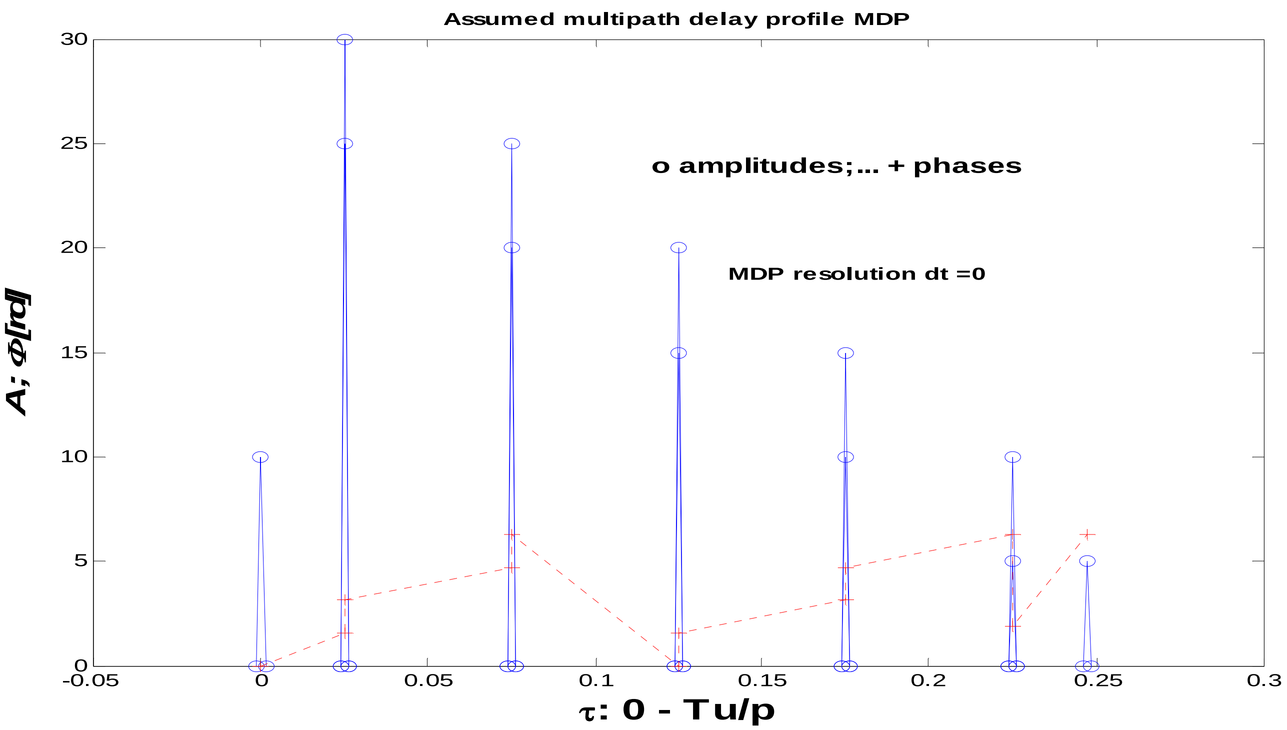

Fig. 19b. MDP profile with minimal assumed time delay differences

Fig. 19a, b. Multipath delay profiles for time resolution estimation

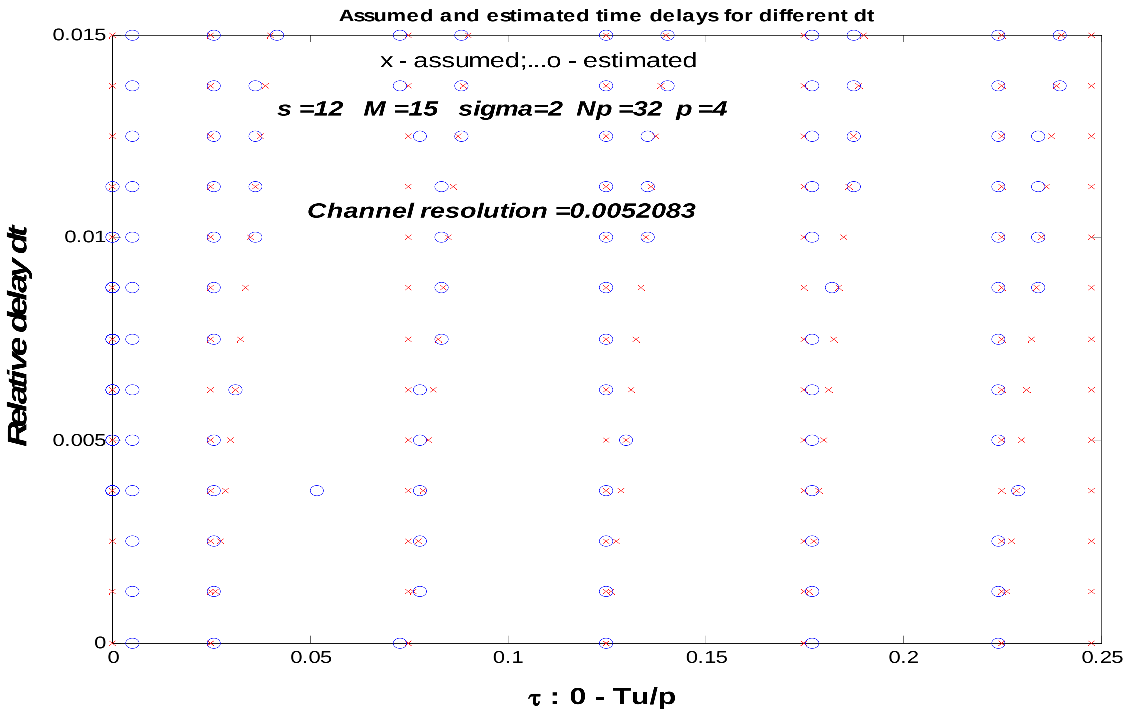

Fig. 20 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different Δτ

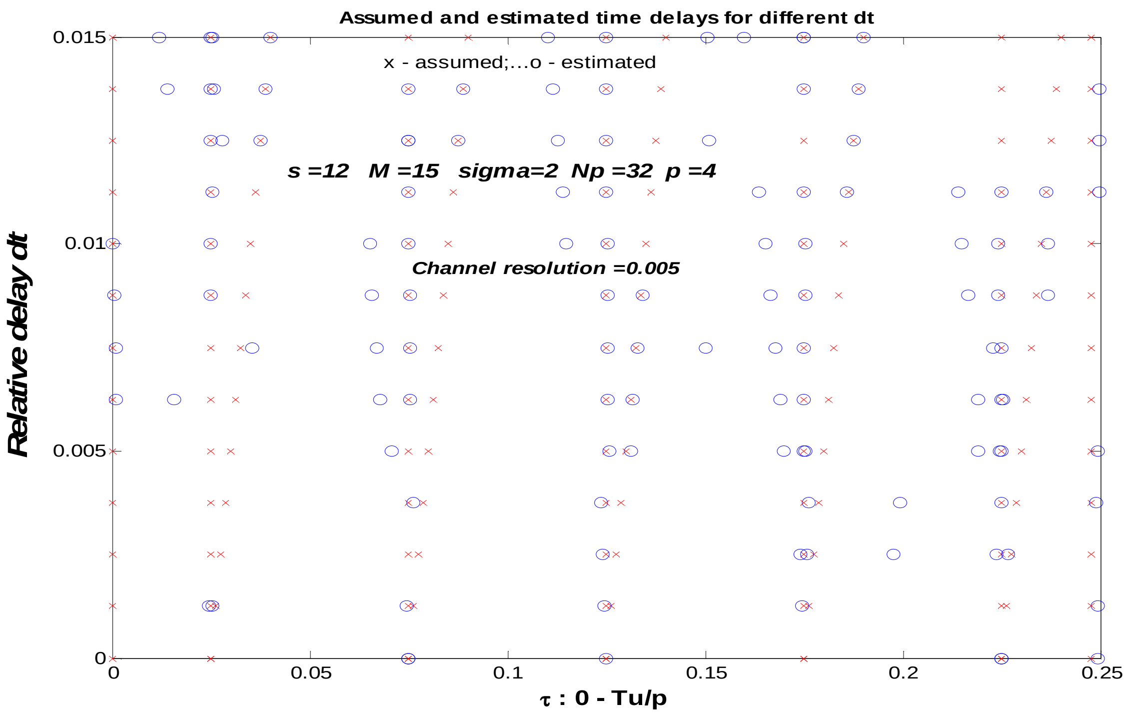

Fig.20 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different Δτ

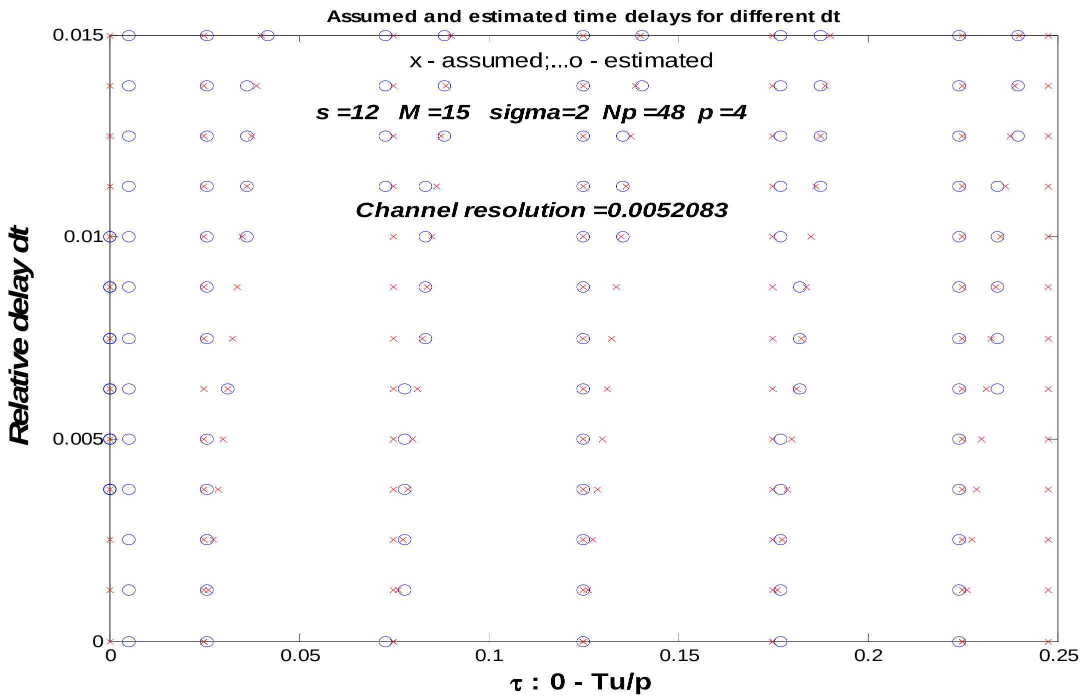

Fig. 21 A. MUSIC. Comparison of assumed (x) and estimated (o) time delays for different Δτ

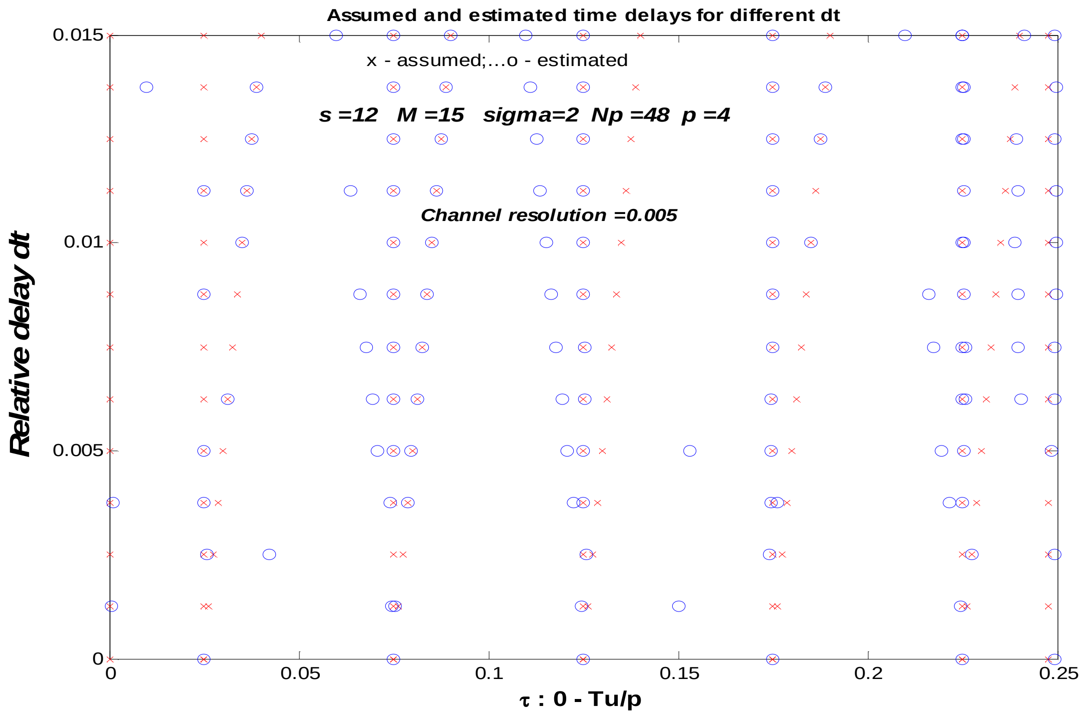

Fig. 21 B. Root-MUSIC. Comparison of assumed (x) and estimated (o) time delays for different Δτ

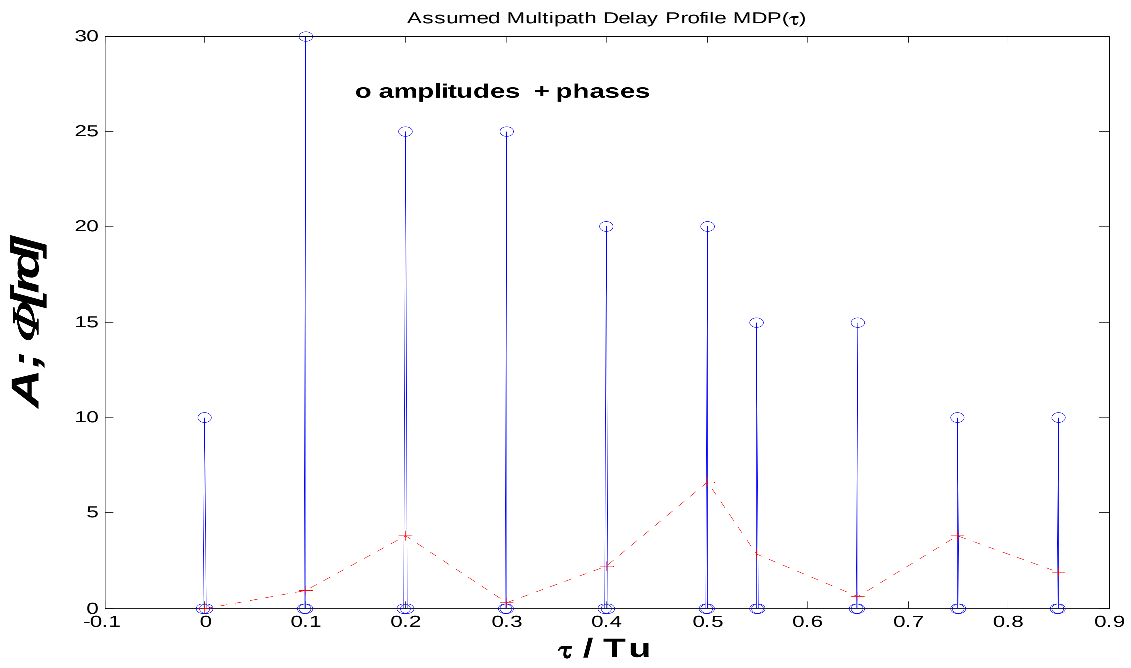

Fig. 22. Multipath delay profile for analyzing MUSIC when MDP include paths with τ > Tu/p

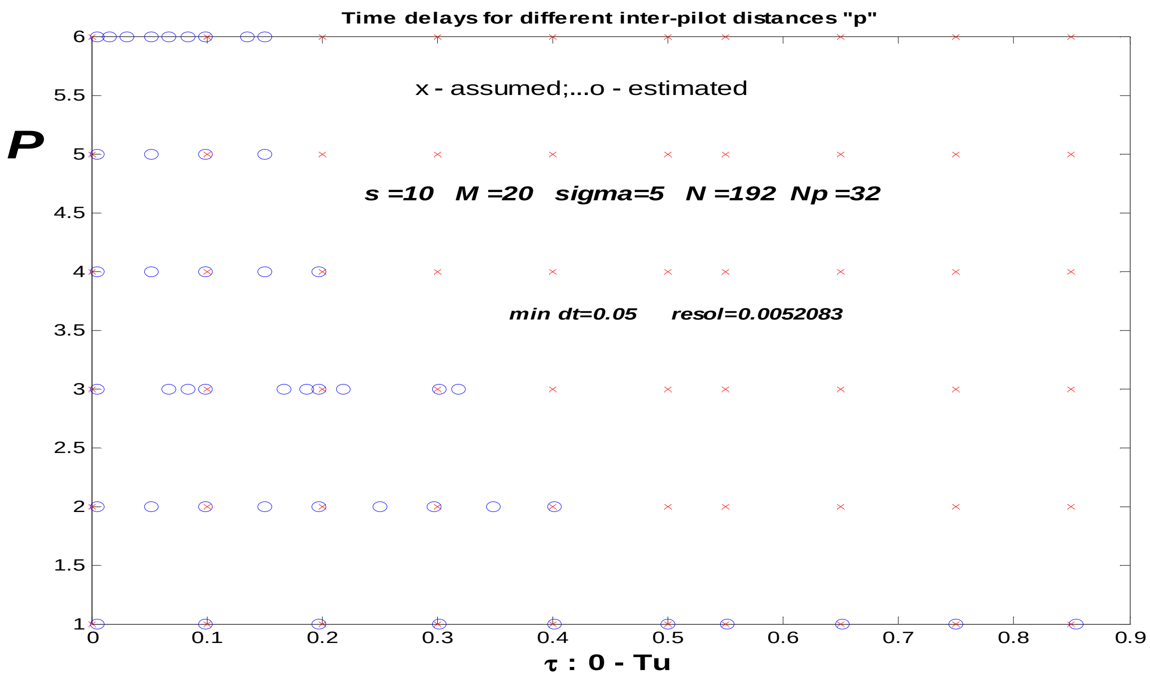

Fig. 23. Comparison of assumed (x) and estimated (o) time delays for different inter pilot spacing p