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Ground-wave propagation
Transmission loss
Fourth-power law

VALIDATION OF THE FOURTH-POWER LAW IN RADIO PROPAGATION

Transmission-loss vs distance relations are confronted with analytic and empirical data. The fourth-power law holds in various terrains, in a three-decade frequency interval, up to the redefined radio horizon. Elementary transmission-loss formulas are accurate in regular open terrains below ca 100 MHz. General formula for irregular terrains is less accurate, but valid in larger frequency and distance ranges.

1. INTRODUCTION

Propagation of useful or interfering radio waves above earth depends, in general, upon various properties of the ground and the atmosphere. According to the predominant factor, the terminology "ground waves" or "tropospheric waves" is / loosely / used. With any mode of over-earth propagation, at distances up to the radio horizon the same fundamental relationship is exhibited: approximate proportionality of mean transmission loss / L / to the fourth power of distance / d /. The $L \propto d^4$ relation, emerging also from Norton's analysis of ground waves over plane earth, has been raised to the status of the "fourth-power law" / FPL /.

Originally, the FPL has been formulated for flat open terrains. However, ample experimental evidence confirms the validity of FPL - in the average sense - in various terrains, even with considerable scattering of radio waves / e.g. in urban or hilly areas /. With respect to distance and frequency, the FPL as such / and often the formulas based upon it / exceeds considerably the originally conjectured ranges of validity.

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In radio engineering, the FPL is implemented with formulas for estimating field-strength or transmission-loss between two antennas. Historically, first expressions by Vvedensky, by Van der Pol, and by Bullington [1], pertained only to propagation above plane smooth earth. Later contributions [2, 3] cope also with earth curvature and typical terrain irregularity, associated by frequency-dependent excess loss.

Unfortunately, authors of the above-cited and similar applications of the FPL did not pay due attention to the validity of the law. In most cases, partially valid formulas were constructed, pertaining only to one specific propagation mode, with indeterminate distance - and frequency-range.

In order to fill in these gaps, the authors of this paper initiated a long-term research program on the FPL. First, see [4, 5, 6], a consistent set of known and novel transmission-loss formulas has been compiled, for typical terrains up to the / redefined / radio horizon. Two most fundamental formulas are shown in Table 1 in decibel notation: the elementary, frequency-independent one and the generalized, frequency-dependent one. The application ranges of the formulas, as conjectured by these authors, are also cited and will be validated below by analysis and experiment.

Table 1. Principal formulas based upon the fourth-power law

Basic transmission loss [dB]	$1 < f < 100$ MHz /elementary FPL/	$120 + 40 \lg d - 20 \lg h_1 h_2$
	$100 < f < 1000$ MHz /generalized FPL/	$100 + 10 \lg f + 40 \lg d - 20 \lg h_1 h_2$
Distance range of validity d_n [km]	$4,0 (\sqrt{h_1} + \sqrt{h_2})$	
Height range of validity $h_{1,2\max}$ [m]	$2700 f^{-2/3}$	

In the Table, h_1 and h_2 denote effective / corrected / transmitting resp. receiving antenna heights. Distance d [km] and frequency f [MHz] are expressed in practical units.

2. GENERALIZED CONCEPT OF THE RADIO HORIZON

At the very start of our research it was noted that the validity of the FPL, with respect to distance between the antennas, exceeds by far the limits suggested in some textbooks and papers. With these sources, the FPL - treated in the light of Norton's theory of ground wa-

ves - is believed to be valid up to the "plane-earth boundary". The actual range turns out to be much broader.

The upper ^{*/}boundary of the FPL relation is exhibited in empirical data [e.g. 7, 8, 9] and propagation curves [e.g. 10, 11] : Our attention was paid to the pure-diffraction mode and to the interrelated notion of the radio horizon: A novel generalization of this concept, as introduced by these authors in [6], appears to roughly coincide with the distance range of the FPL.

The classical radio horizon, controlled by the refraction of space waves in the atmosphere, is located at the / standardized / distance:

$$d_r = 4,12 (\sqrt{h_t} + \sqrt{h_r}) \quad [\text{km}] \quad / 1 /$$

Here, h_t and h_r stand for the actual heights of the transmitting resp. receiving antenna in meters over ground. With our approach, the radio-horizon notion is extrapolated to all kinds of ground/tropospheric waves and related to FPL-validity. A novel formula analogous to / 1 / is introduced as a general estimate of the radio-horizon distance:

$$d_h \cong 4,0 (\sqrt{h_1} + \sqrt{h_2}) \quad [\text{km}] \quad / 2 /$$

where $h_{1,2} = \sqrt{h_0^2 + h_{t,r}^2}$ denote the effective antenna heights, corrected with h_0 , the virtual minimum antenna height, cf [1, 6] :

In the special case of space-waves above earth, actual antenna heights are inserted into / 2 /:

$$d_h \cong 4,0 (\sqrt{h_t} + \sqrt{h_r}) \quad [\text{km}] \quad / 3 /$$

In eq. / 2 / and / 3 / the numerical coefficient - 4,0 with two decimal digits - reflects the limited accuracy of the radio-horizon estimates:

With surface waves in the HF range, h_1 and h_2 reduce to h_0 and / 2 / reduces to:

$$d_h \cong 8,0 \sqrt{h_0} \quad [\text{km}] \quad / 4 /$$

Note that the classical plane-earth criterion, dependent on frequency only [6], is replaced by / 4 / related also to electrical parameters of the ground - as it should be with surface waves.

Novel generalized measure / 2 / of the radio-horizon distance stands in Table 1 for the range of validity of principal transmission-loss formulas derived from the FPL. This conjecture will be validated below.

^{*/}The lower limit, imposed by the near-field distortion, is not of concern here.

3. VALIDATION OF THE PROPAGATION LAW

A vast amount of experimental data on radio propagation over earth are available in published literature, but many authors did not care to inform exhaustively about the conditions of experiments. Seemingly, if only the existence of the propagation law is searched, bare field-strength-vs-distance curves are sufficient. However, for comparing the distance-range of the law with the radio horizon, all parameters implicit in /2/ are necessary. In preparing this paper, a catalogue of well-documented sources was made and is being kept by the authors. Data were collected in a possibly broad range of frequencies, subdivided into 2 parts in agreement with Table 1; the population of terrains is duly diversified.

In our preliminary research, the FPL - emerging from analysis as a proportionality relation $\bar{L} \propto d^4$ - was confronted with published empirical data. The accuracy of transmission-loss formulas was initially disregarded. In Fig.1 and Fig.2 results of measurements, converted into transmission-loss values if necessary, are represented by points or by curves fitting to the data. The abscissas express the relative distance, normalized by the radio-horizon distance as calculated /by these authors/ with /2/. The ordinates show the deviation of the data $/L_{exp}/$ from the FPL reference, i.e. the horizontal straight line. The latter is separately fixed for any experiment with steady conditions by determining the trend in data points at shorter distances.

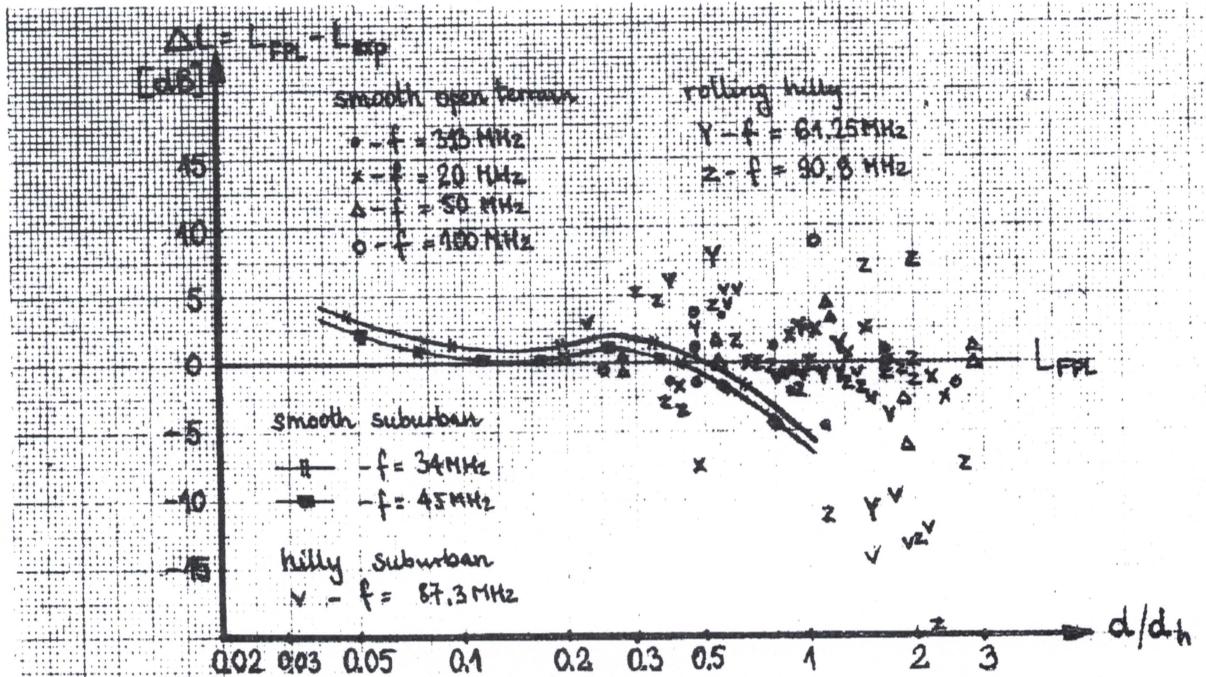


Fig.1. The fourth-power law reference $/L_{FPL}/$ vs empirical data $/L_{exp}/$ in the frequency range up to 100 MHz.

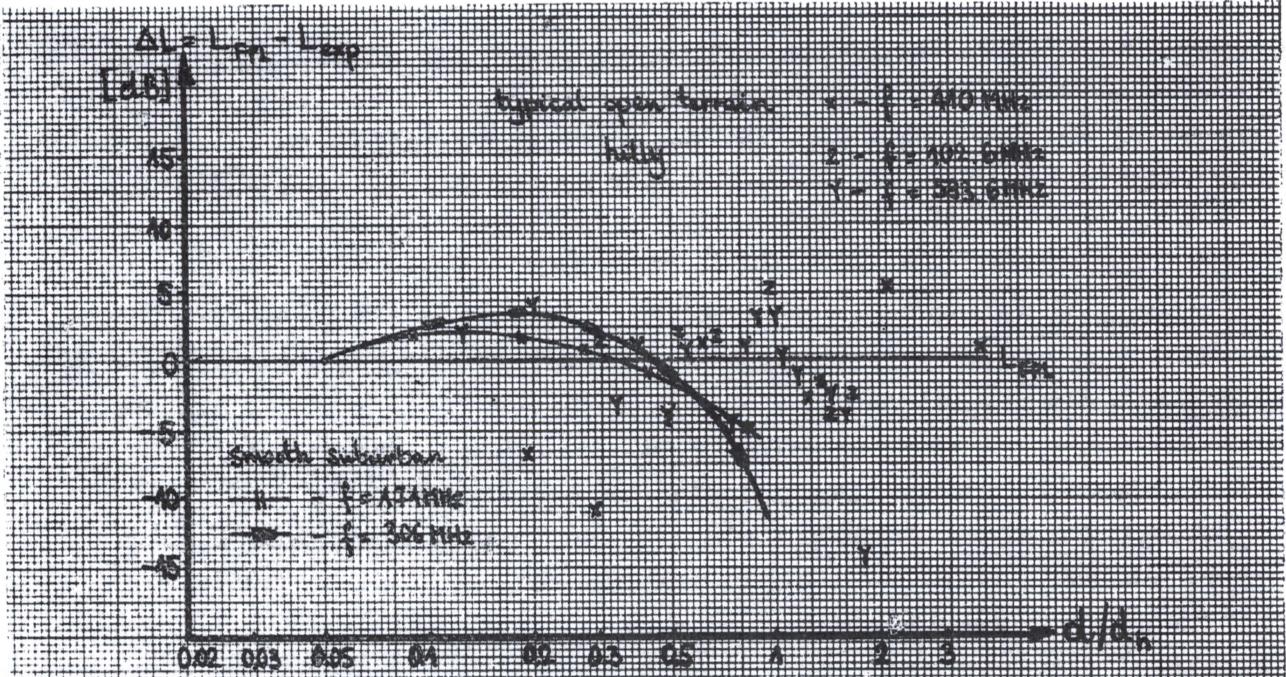


Fig.2. The fourth-power law reference L_{FPL} vs empirical data L_{exp} in the frequency range 100+1000 MHz.

In urban terrains, the random multipath propagation exhibits very large location variability and the FPL holds true only in the average sense [cf [5,6]]. As the horizon distance is rather meaningless, in Fig.3. the abscissa expresses the original distance. In order to cope with immense spread of data, envelopes of large sets of /e.g.200/ points are compiled, cf [12]. The best-fit straight lines represent again the FPL reference for any data-field. These reference lines convey implicitly the information about median /distance-independent/ scattering loss in urban terrains.

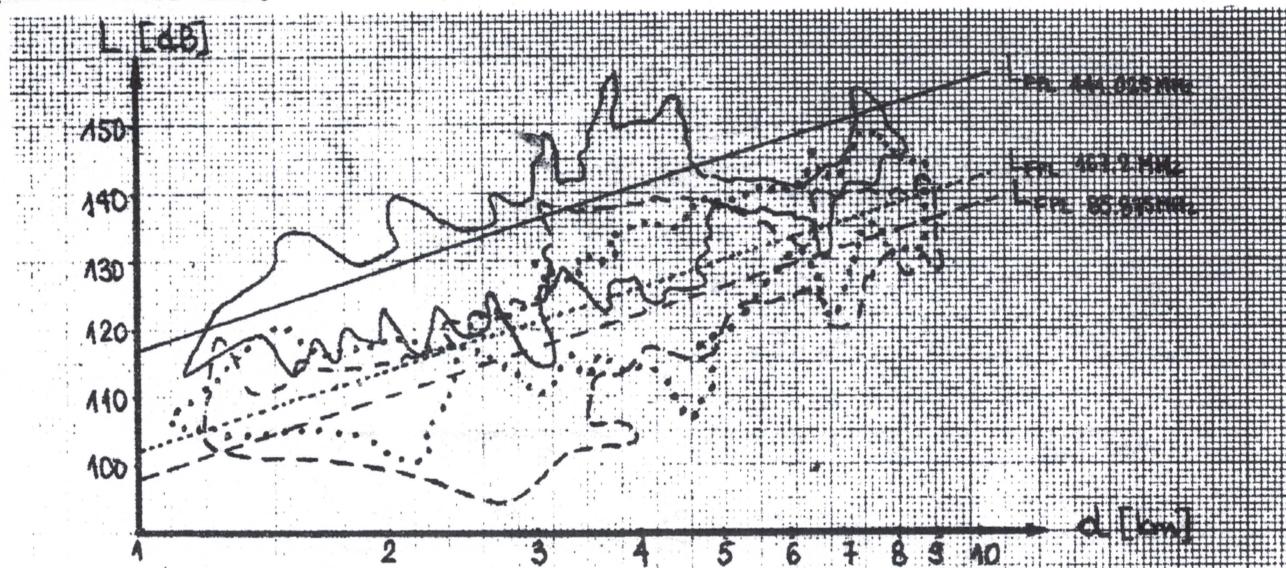


Fig.3. Median transmission loss in urban terrains. The FPL reference straight lines vs envelopes of dispersed empirical data.

Extensive investigation of empirical data, partly shown in Figs.1-3, clearly leads to the important conclusion: the FPL exists objectively with over-earth propagation in various terrains, in the large frequency interval of ca three decades. In regular open terrains, the distance range of validity of this law is indicated by the generalized radio horizon.

4. VALIDATION OF FORMULAS BASED UPON THE FPL

A generally formulated propagation law does not necessarily imply the existence of analytic propagation formulas. As already stated, simple analytic transmission-loss formula in Table 1, as well as its counterparts in [6], apply only to regular open terrains. The generalized formula in Table 1, and a similar proposal in the EMC'80 paper [13], are of semiempirical origin. Formulas of both types were confronted by these authors with analytical and empirical results.

When radio waves propagate over smooth spherical earth the field strength at any frequency and distance can be computed exactly. Non-closed-form diffraction formulas by Van der Pol and Bremmer are an excellent analytical tool for validating the accuracy and the distance range of elementary FPL-based formulas. An example of how some approximate results fit the exact calculations is presented in Fig.4. The frequency range /up to 300 MHz only/ reflects the relative roughness of any longer UHF radio path /over land/.

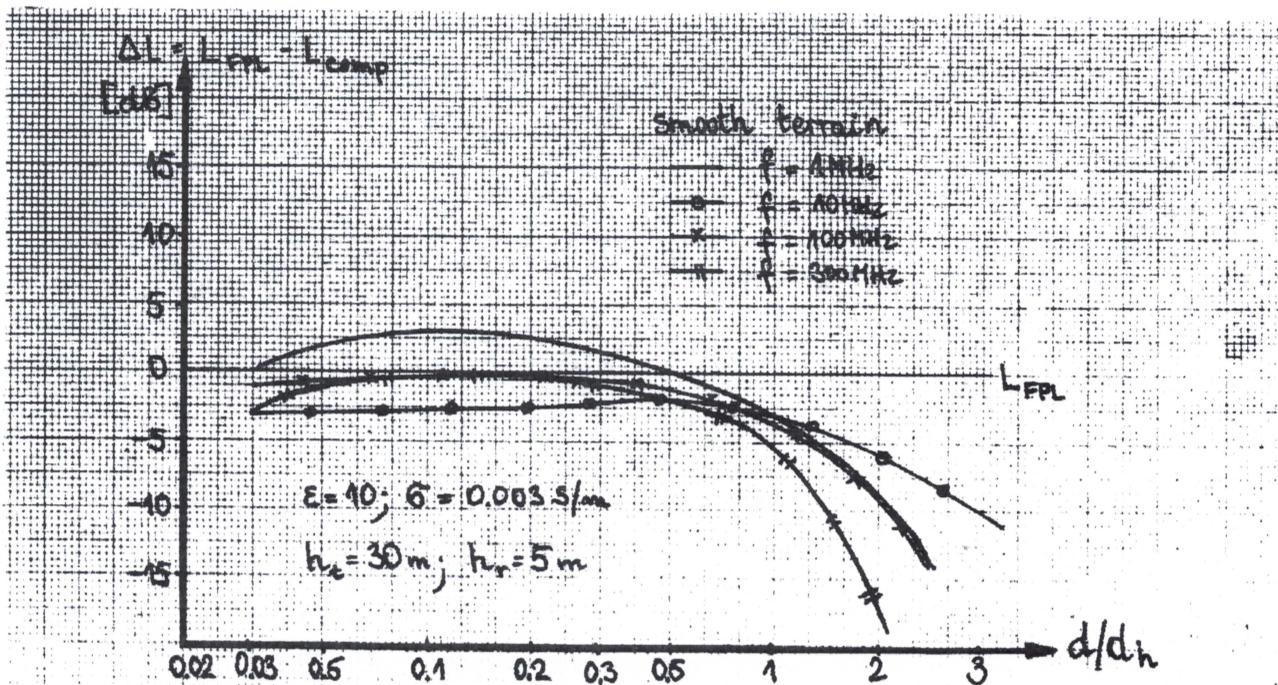


Fig.4. Calculated transmission loss over smooth earth. Comparison of exact results L_{comp} with first-order approximations.

Large-scale comparisons /not shown here in detail/ convince uniformly that elementary FPL-formulas do approximate, with engineering accuracy, the transmission loss on smooth HF/VHF radio paths within the radio horizon.

In empirical validation of various FPL-originated expressions, compliance between conditions of experiments and legitimate application ranges of formulas is needed. Some data useful for proving the existence of the law, like those from urban terrains /Fig.3/, have no counterpart in mathematical form.

Data extracted in this regard serve in Figs.5 and 6 for verifying the accuracy and the distance range of formulas in Table 1. As the difference between calculated and measured transmission loss is plotted vs relative distance, conjectured coincidence between radio horizon and the range of validity of the formulas can be verified.

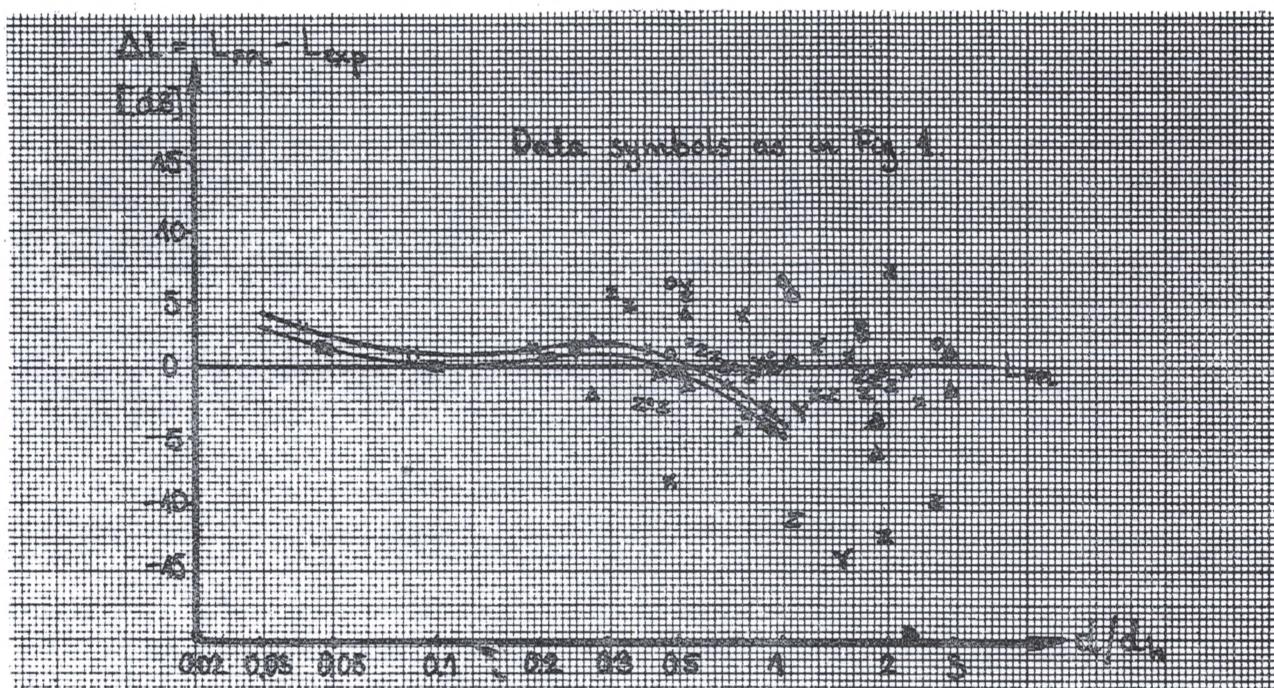


Fig.5. Formulas based upon FPL vs experimental data /frequency range 1+100 MHz/.

Comprehensive work, of which some examples only are here presented, validates the FPL estimates within the accuracy of a few decibels, up to the radio horizon. The elementary, frequency-independent formula holds in open terrains up to about 100 MHz. The novel generalized formula /above 100 MHz/ takes care of terrain-affected scatter and cannot be equally accurate in different terrains. With diminished accuracy /of several decibels/ its validity exceeds the radio-horizon, as could be

expected in irregular terrains. It is reminded that only average values obey the fourth-power law.

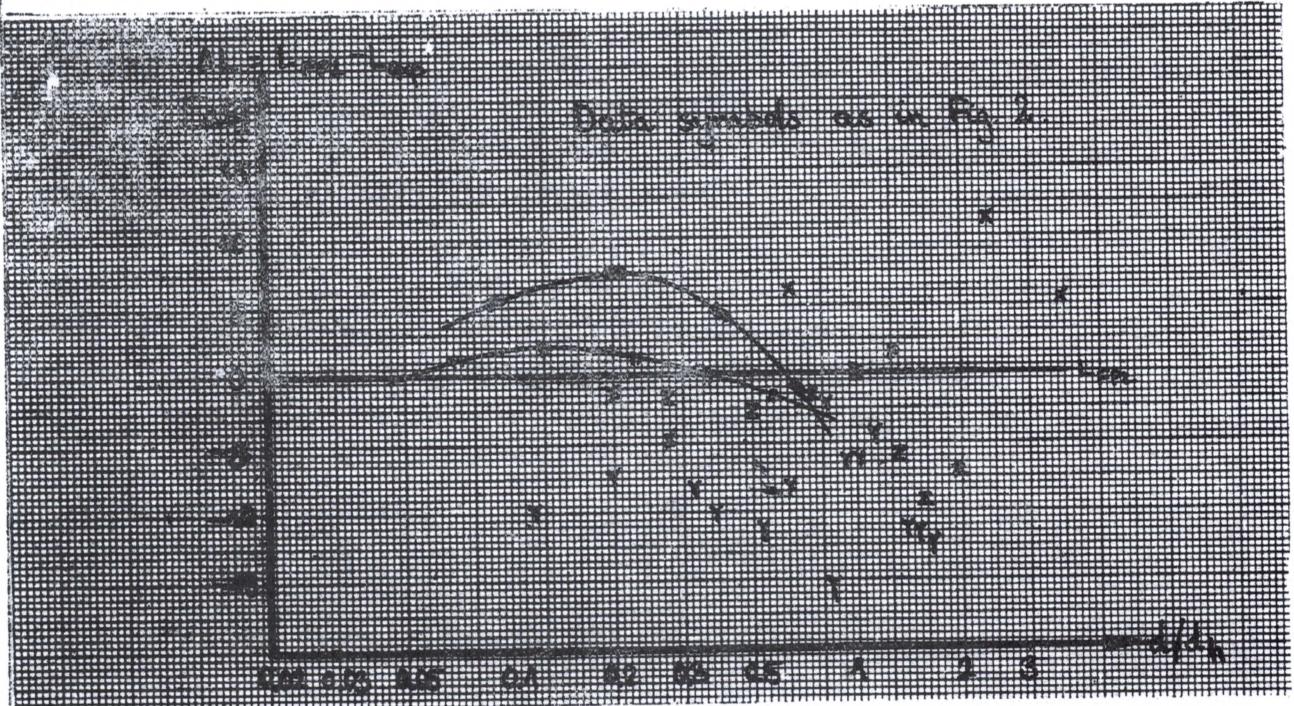


Fig.6. Formulas based upon FPL vs experimental data /frequency range 100-1000 MHz/

5. GENERAL CONCLUSIONS

In this paper, FPL transmission-loss vs distance relationships were confronted with analytic and empirical data. It is found that the fourth-power law holds in various terrains, in a nearly three-decade frequency interval, up to the redefined radio-horizon. Elementary analytic transmission-loss formulas /based upon the FPL/ are accurate in regular open terrains below ca 100 MHz. Novel generalized /semiempirical/ formula for slightly irregular terrains is less accurate, but is valid in larger frequency- and distance ranges.

By and large, the fourth-power law can be considered as validated by analysis, computations and experiments. At higher frequencies and/or in irregular terrains, validity of the law is understood in the average sense.

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Проверка закона четвертой степени в распространении
радиоволн

При распространении радиоволн над землей, вплоть до радиогоризонта, употребляется одна общая зависимость: приближенная пропорциональность средних потерь передачи до четвертой степени расстояния. Эта общеизвестная реальность получила статус закона четвертой степени /ЗЧС/. Первоначально сформулирован для открытой, плоской местности, он кажется быть действительным – в усредненном смысле – также для шероховатых местностей, даже для городской застройки. Найдено, что ЗЧС действует в около трех декадах с 1 до 1000 МГц. С другой стороны ЗЧС и его действительность иногда неверно понимают для полезных и мешающих сигналов.

Чтобы установить реалистические критерии действительности для ЗЧС и связанных формул, была поставлена исследовательская программа. Сравнивая большие количества экспериментальных данных и проводя вычисления на ЭВМ, были внимательно рассмотрены следующие вопросы:

- объективное существование и характер закона,
- области действительности ЗЧС и связанных формул,
- действительность в области расстояний, в связи с новой концепцией радиогоризонта,
- известные и новые формулы на потери передачи, основанные на ЗЧС.

В согласии с уже опубликованными частичными результатами, наши окончательные выводы подтверждают что:

- ЗЧС является объективно существующим физическим законом,
- действительность ЗЧС значительно шире, чем принимается это в литературе,
- закон действителен для разных местностей и в трех частотных декадах,
- является обоснованным пересмотр понятия радиогоризонта,
- пересмотренный радиогоризонт может рассматриваться как расстояние предела действительности ЗЧС и связанных формул,
- элементарная формула для потерь передачи является точной для ровных, открытых местностей в диапазоне частот до 100 МГц,
- является обоснованным ввести новые формулы для распространения, основанные на ЗЧС и учитывающие рассеяние от местности,

- обобщенная формула для потерь передачи менее точная, но она действительна в расширенном диапазоне частот и расстояний.

Рис. 1. ЗЧС L_{FPL} в сравнении с экспериментальными данными L_{exp} в диапазоне частот до 100 МГц,

Рис. 2. ЗЧС L_{FPL} в сравнении с экспериментальными данными L_{exp} в диапазоне частот от 100 до 1000 МГц.

Рис. 3. Медианные потери передачи для городских местностей. Прямые ЗЧС в сравнении с расбросанными экспериментальными данными.

Рис. 4. Вычисленные потери передачи над гладкой землей. Сравнение точных результатов L_{comp} с приближением первого порядка L_{FPL} .

Рис. 5. Формулы основанные на ЗЧС в сравнении с экспериментальными данными /диапазон частот I до 100 МГц/.

Рис. 6. Формулы основанные на ЗЧС в сравнении с экспериментальными данными /диапазон частот 100 до 1000 МГц/.